Current Meter Calibration Equations:
A review of the adequacy of limited-range equations for New Zealand requirements

NIWA Client Report: CHC2008-144
October 2008

NIWA Project: ELF09205

Current Meter Calibration Equations:
A review of the adequacy of limited-range equations for New Zealand requirements

Dave Johnstone

Prepared for

Tasman District Council

NIWA Client Report: CHC2008-144
October 2008
NIWA Project: ELF09205

National Institute of Water & Atmospheric Research Ltd
10 Kyle Street, Riccarton, Christchurch
P O Box 8602, Christchurch, New Zealand
Phone +64-3-348 8987, Fax +64-3-348 5548
www.niwa.co.nz

Contents

Reviewed by: Approved for release by:

Graeme Horrell Ross Woods

Executive Summary

Regional Councils and other agencies use current meters to measure water velocity, and hence river flows. This information is a core part of water allocation and management, and is also used for a wide variety of engineering, recreational and scientific purposes. Most current meters used for streamflow measurements in New Zealand are calibrated at the National Institute for Water and Atmospheric Research Ltd (NIWA). The NIWA calibrations are expressed as linear equations, applicable to recommended ranges of velocity which are specific to the meter type. In the course of low flow stream gaugings, it is commonly necessary to make velocity measurements outside of the recommended ranges. This study shows that systematic errors will occur when velocities are calculated by significant downward extrapolation of the linear calibration equations, outside the recommended range. These errors can be large for all meter types. Curvilinear calibration equations are shown to be capable of more accurate prediction of low velocities, when compared to extrapolation of linear calibration equations. Recommendations are made for changes to NIWA's current meter calibration procedure.

1. Introduction

Mechanical current meters have been used for river flow measurement for over 100 years, and they will continue to be the instrument predominantly used for this purpose for the foreseeable future.

There are two types of current meter: horizontal axis propellor/fan types and vertical axis bucket wheel types. Both types are used in NZ. Some models of propellor type meters have optional propellers intended for different velocity ranges. In this document, each propeller for a particular model is referred to as a "meter type". An extensive review of literature from current meter studies, for both vertical axis and horizontal axis meters, was carried out by Thibodeaux (1994).

Throughout the world, calibration of current meters is carried out by towing meters along a tank at a series of predetermined speeds. NIWA has a purpose-built 50 m long tank at Kainga, near Christchurch. Most of the current meters used for flow measurements in NZ are calibrated at this facility which has been in operation for approximately 50 years. The rotation rate or output of a meter, (revs/sec), for each velocity is signaled by the rate of switch closures produced by the meter. The NIWA rating car can operate reliably over the velocity range: 0.025 – 3.2 m/s.

At the USGS HIF current meter rating facility, meters are towed in both directions at each of several nominal velocities, according to Hubbard et al. (1999). The calibration data from both directions are averaged to ensure they are not affected by any currents that may exist in the tank. The USGS uses standard ratings for batches of Price AA and Pygmy meters, having found that an average (standard) rating gives nearly as accurate results as individual ratings. As an outcome of a study of the quality of velocity data from standard ratings by Hubbard et al. (1999), the USGS identified a need to reduce current meter errors arising from the use of standard ratings. Environment Canada also principally use the Price AA and Pygmy meters, but feel it necessary to rate each meter individually; see DeZeeuw and Bil (1975). A variety of vertical axis and horizontal axis meters are used in New Zealand, and are calibrated individually by NIWA.

This review of the NIWA current meter calibration practice, particularly the mathematical expression of calibrations, and of international practices, follows on from a current meter calibration workshop held in Christchurch in 2002. The workshop was jointly organized by Regional Councils and NIWA, and attended by people from Regional Councils, NIWA and other interested New Zealand agencies, and also by representatives from US and Australian agencies that calibrate current meters.

The 2002 workshop identified the following limitations with the existing calibration process:

- Calibrations for all meters are represented by linear equations. While some meters have a linear response for part of their range, none can accurately be characterised by linear segments throughout their range, especially the low velocity range.

- Much of the reason for this was the historical difficulty of fitting curves to calibration data, and applying non-linear calibrations to gauging data. This has now changed.

- Currently different propellers must be used for different velocity ranges, to comply with the limited ranges allowed for NIWA calibrations. A single curvilinear calibration equation, or a series of linear equations, would allow one propeller to cover a much wider range. This is particularly relevant to the small Ott and Oss Prop 1 current meters.

- The meeting identified a need for research into an improved calibration practice, a more detailed calibration report, and a change to existing gauging software so that the new calibration equations could be implemented.

Since 2002 NIWA has introduced a new format, more detailed calibration report which shows the meter mounting method, the position of the meter in the tank cross-section, the velocity and rotation rate for all the calibration points, and the deviation of calibration points from the fitted line.

The objective of this further review is to assess the adequacy of NIWA's single straight line, limited range calibration expressions for gauging requirements in New Zealand, and to consider whether the adoption of extended range calibration equations, either single curvilinear equations, or a series of equations, would provide an advantage.

The manual method widely used to derive a set of straight line equations to represent a calibration is described in ISO 3455:2007, and is summarized below. The procedure is also described by Herschy (1985), and in an early US Department of Commerce report, Johnson (1966). An undated Ott publication held by NIWA describes the procedure specified in ISO 3455.

An example calibration certificate supplied to NIWA by the Swiss National Hydrological and Geological Survey in 1999 depicts the ISO 3455 procedure. The ISO specifications for current meter calibration were originally developed from experience at the Swiss facility. Documents entitled “Calibration Laboratory” and “Calibration laboratory for hydrometric measuring instruments”, both by Samuel Graf (2002), of the Swiss National Hydrological Survey, describe the Swiss calibration equipment and procedure. Where possible, the Swiss facility takes a series of 10 measurements for one calibration point. The mean of these is calculated and the result used as the calibration point.

Ott, the Swiss agency, and an Australian agency, all use multiple straight line representations for some meters. The USGS developed new standard rating tables for Price AA and Pygmy meters in 2001 Hubbard et al. (2001), and changed from use of two linear segments, describing upper and lower ranges of velocity, to use of a single straight line calibration equations for the Price AA and Price Pygmy meters.

The ISO 3455 procedure consists of the following steps: The calibration points are plotted on a large piece of graph paper. A line shown plotted through a set of calibration points takes the form shown schematically in Figure 1. Velocity, V, is plotted on the ordinate, and the rotation rate, n, is plotted along the abscissa. Conspicuously spurious readings are discarded. In practice the calibration line appears very close to a straight line. Ideally, in the absence of friction, switch detention, and disturbed backwater from the support, the calibration line would be a straight line passing through the origin, shown as the dashed line in Figure 1. The equation for this line has the form:

$$V_0 = k_0.n$$

Where V_0 is velocity (m/s), k0, the slope is equal to the geometric pitch of the propeller (m/rev), and n is the rotation rate (revs/s).

A real calibration line presents itself as a flat hyperbola starting a small distance up the ordinate, corresponding to the starting speed of the meter, and converging at higher velocities with an asymptote that is parallel to, or equivalent to a straight line passing through the origin.

In accordance with the ISO 3455 procedure, a second “expanded curve”, shown schematically in Figure 2, is plotted. ΔV is the difference between the observed velocity V, and the velocity determined by the assumed asymptote V_0 shown as the dashed line in Figure 1. The difference is written

$$\Delta V = (V - V_0)$$

Substituting k_0n for V_0 gives

$$\Delta V = (V - k_0n)$$

as the ordinate of the expanded rating curve. The expanded rating curve can be estimated as one or more straight line segments shown as A, B, and C in Figure 2, whose equations take the form

$$V - k_0n = an + b$$

Where a is the slope and b is the intercept. Solving for V, the corresponding calibration equations have the form

$$V = (k_0 + a)n + b$$

Substituting c for the constant representing the slope $(k_0 + a)$, the form for the three rating curve equations becomes

$$V = cn + b$$

The range of n for which each equation is valid must be specified.

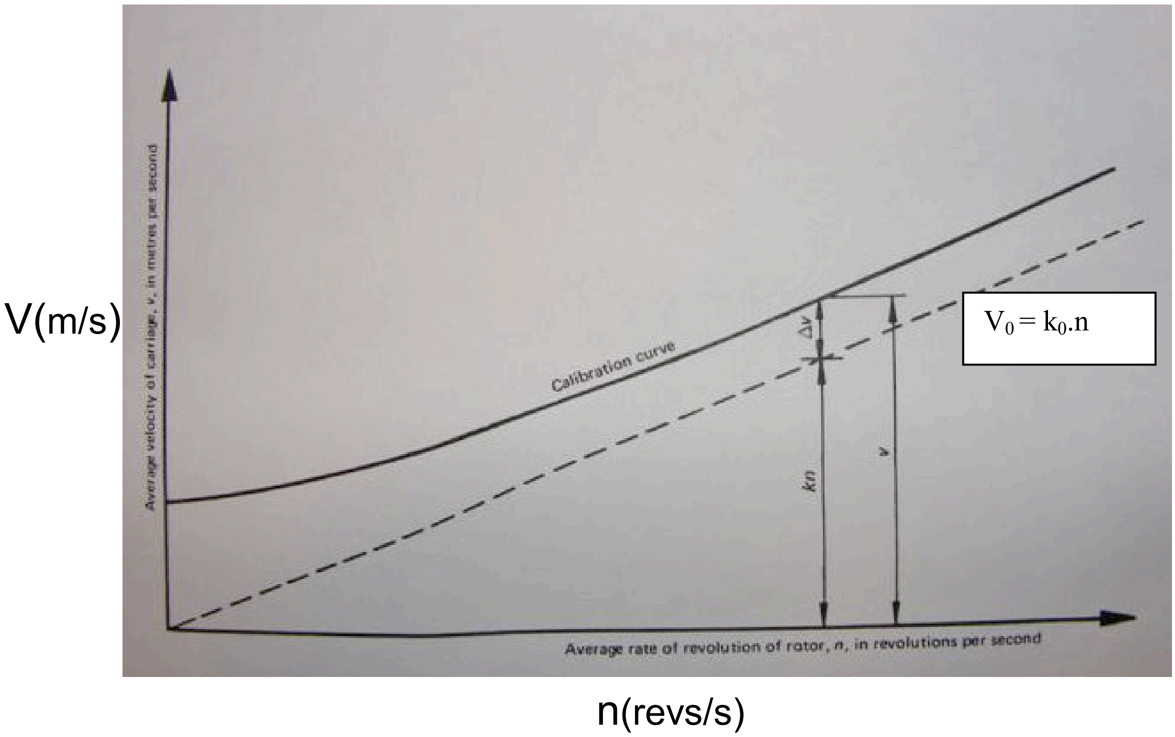

Figure 1: Schematic of current meter rating curve and assumed asymptote

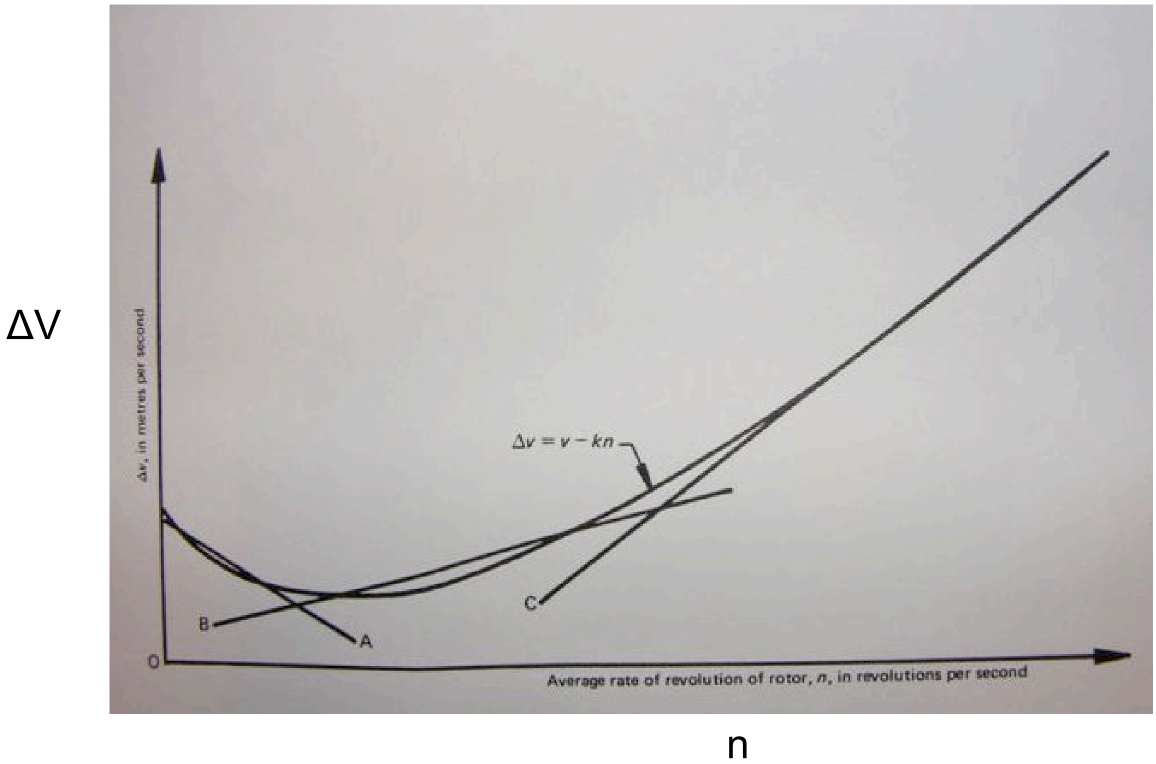

Figure 2: Schematic of expanded current meter rating curve

Current meter calibrations carried out at NIWA's tow tank have been expressed as a single straight line, with a recommended limited applicable range. For the last decade or so, NIWA has used the least squares method to fit a straight line to set of calibration data.

There is a risk that a spurious calibration point will not be detected when a computer is used to fit a line to the calibration data. Since 2002, NIWA has used "sensitivity curves", described below, to visually highlight any spurious calibration points, which can then be repeated or discarded.

Calibration tables can be produced, allowing velocity to be read from a table at the time of gauging. More commonly nowadays, velocity is calculated from the applicable calibration equation on a gauging computer at the time of gauging, or on a PC following the gauging.

Engel (1989) and Engel and Weibe (1993) have shown that linear equations do not provide an adequate fit to calibration data for Price meters for velocities less than 0.03 m/s, where the rotor response is decidedly nonlinear. Engel (1999) uses plots of n/V (rev/m) versus V to highlight the nonlinearity of calibration curves at low velocities, and concludes that linear calibration equations should not be used if low velocity accuracy is important. A single continuous calibration equation that combines the linear and nonlinear components of rotor response for Price meters was developed by Engel (1989). That equation is

$$V = an + be^{-kn}$$

Where V is velocity (m/s), n is rotation rate (rev/s), and a, b, and k are coefficients to be determined.

2. 2002 Workshop outcomes, conclusions and recommendations

Material presented and discussed at the 2002 workshop that is relevant to this review of NIWA practice is reported here.

A report, "Current Meter Behaviour and Calibration Equations", Johnstone (2002) discusses two distinct states manifested by current meters over extended ranges of velocity for most meter types (exceptions are the small Ott and Oss, prop 1 meters) (Figure 3). For the upper velocity range, the relationship between velocity and the number of meter rotations for a given water translation distance becomes constant, and independent of the rate of translation (water velocity), analogous to the movement of a screw in a medium such as wood. The upper velocity state is referred to as the "screw state". For the lower "sub-screw" velocity range, the meter is said to be "slipping", and the meter "sensitivity" (n/V, or revs/m), is below that for the screw state. The sensitivity is low for low velocity and increase rapidly as velocity increases. The "sensitivity" plot, n/V versus V, for a meter type, was identified as highlighting meter behaviour, particularly the characteristic increase in meter sensitivity, (revs/m), as velocity increases and for most meter types, a stable revs/m state above a threshold velocity.

Sensitivity curves clearly show that for most meter types, the meter sensitivity, in terms of rev/m, becomes constant at speeds > than approx 0.7 m/s, indicating that in these cases the appropriate representation of this screw state for the rating curve (V versus n) is a straight line with a 0 m/s intercept (Figure 4). To date calibration relationship expressions do not recognize this characteristic of meters, and to a small extent compromise the representation of this stable output range, by including the weighting of calibration points at lower velocities in the derivation of equations that cover this region.

The sensitivity curve also provides clear evidence of any anomalous meter behavior during the calibration that may indicate the need to discard a calibration point, or to check the meter and repeat the calibration.

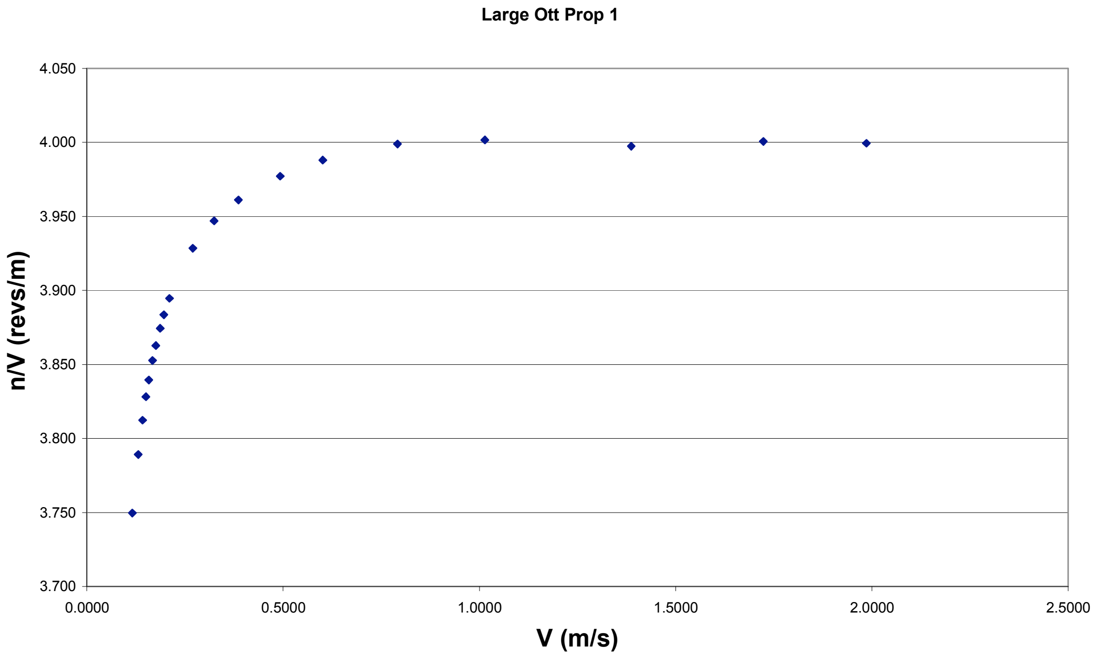

Figure 3: A typical sensitivity curve, in this case for a large Ott prop 1

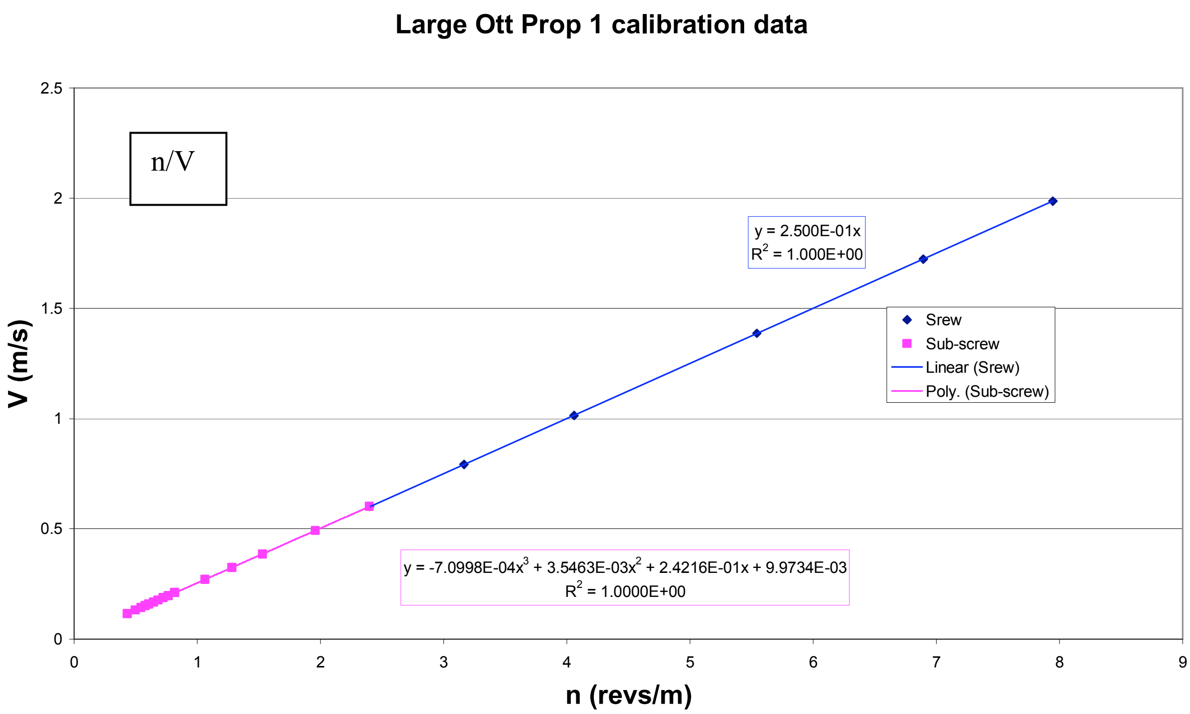

Figure 4: Large Ott prop 1. Straight line zero intercept for screw state, and third order polynomial line for sub-screw state.

This example suggests that a set of V and n calibration points for a meter that attains the screw state could be represented quite accurately as a third order polynomial function for the sub screw state, and a straight line with zero intercept for the screw state.

Propellers 1 and 5 for small Oss and Ott meters do not reach a screw-like state for velocities up to 2 m/s.

Figure 5 is a sensitivity curve for a small Oss prop 1. Data for 5 successive calibration runs are plotted.

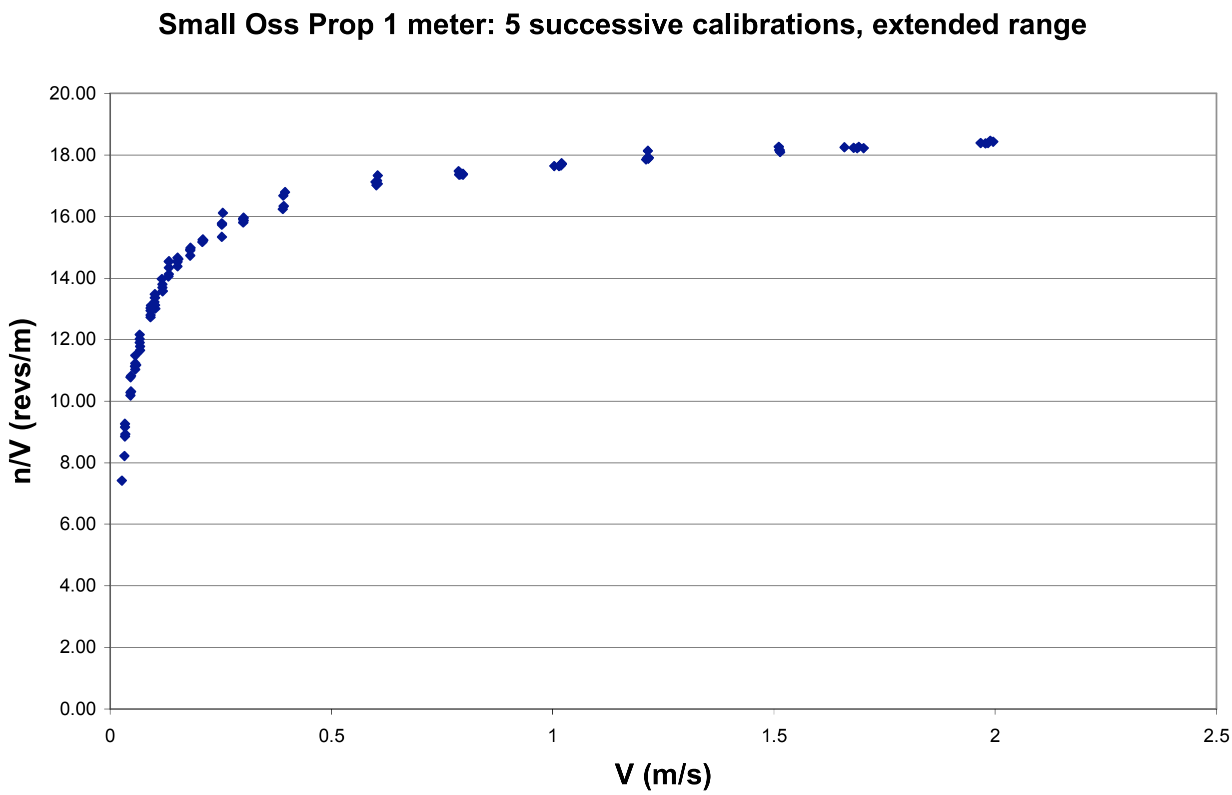

Figure 5: Sensitivity curve for a small Oss Prop 1 meter, 5 successive runs

Figure 5 shows that the output of the small Oss prop1 meter is still increasing with increasing velocity at 2 m/s, and also shows the variability of the meter output over the 5 calibration runs.

Propeller 3 for small Oss and Ott meters does reach a screw state, at quite low velocities (approximately 0.1 m/s). Figure 6 shows the sensitivity curve for a single calibration of a small Ott prop 3.

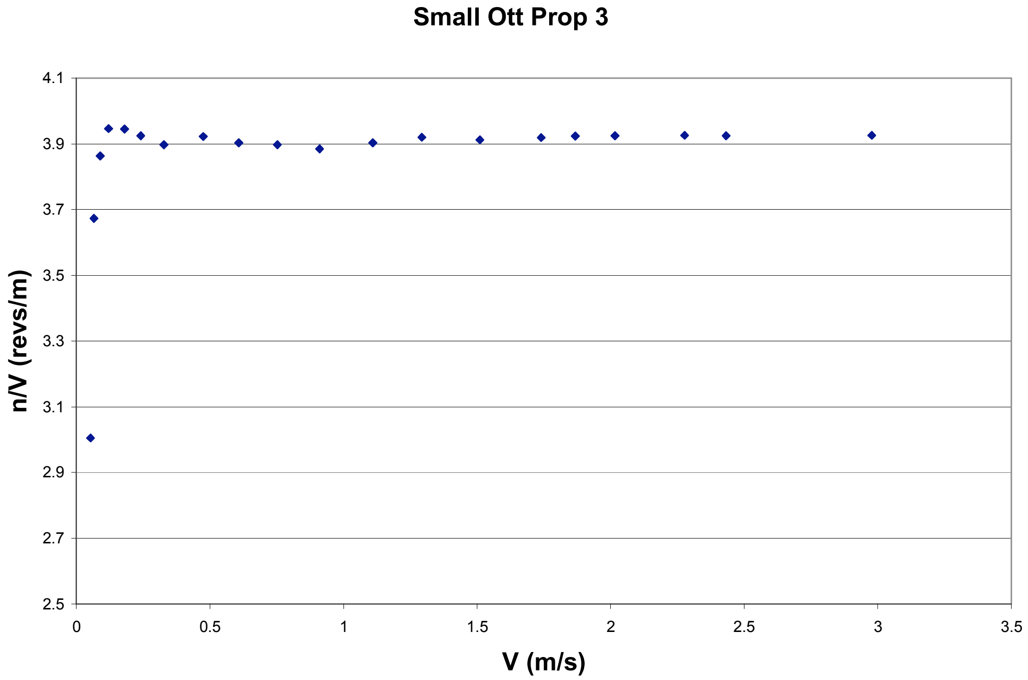

Figure 6: Sensitivity curve for the small Ott Prop 3 meters.

The sensitivity curves for mini Seba props 1 and 3 are similar to those for small Ott and Oss meter props 1 and 3.

For comparison, approximate sensitivity values at 2 m/s for a range of meter types are listed in Table 1. Values at 2 m/s were chosen because sensitivity is relatively stable at this velocity. Sensitivity values can be expected to vary slightly between meters of the same type.

Table 1: Sensitivity, or output for common meters at a velocity of 2 m/s.

Meter		Sensitivity (revs/m)	Effective pitch (m/rev)
Price AA (Read Switch model)		1.5	0.68
Small Ott	Prop 1	18.4	0.05
	Prop 3	4.0	0.25
	Prop 5	18.3	0.06
Large Ott	Prop 1	4.0	0.25
	Prop 2	2.1	0.48
Large Seba	Prop 1	3.0	0.33
Pygmy (Read Switch model)		3.4	0.30
Amsler		3.3	0.31

The small Ott or Oss meters (fitted with the fine-pitched propeller 1) are popular for low flow measurement in NZ, because of their high sensitivity (high revs/m for a given velocity). The NIWA single straight line calibration for this meter limits its recommended use to a small velocity range. Significant systematic errors are incurred if the single line calibration is applied to measurements outside the recommended range (see test results below). Extending the applicable calibration range would remove the necessity to change propellers for accurate measurements outside the recommended range. An additional linear segment, or an alternative expression is required to allow accurate measurements below the recommended range. A non-linear full range calibration expression for this meter, which was proposed at the 2002 workshop, may prove to be best option to allow velocity measurements across a large range, with optimum accuracy.

The meter behaviour as shown by the sensitivity curves is referred to in the study of NIWA calibrations reported here.

At the 2002 workshop a curvilinear "Beta function" was proposed as an alternative to a single straight line, limited range calibration equation for the small Ott and small Oss Prop 1 meters. The Beta function has the form:

$$V = kn + n^d(c-n)^b$$

Where k is the pitch of the meter and a, b, c and d are parameters to be estimated. Fitting three straight lines would require six parameters (three slopes, and three intercepts). A poster paper that McKerchar (2002) presented at the 2002 NZ Hydrological Society symposium describes the application of the beta function.

The fit of the beta function to extended range calibration data for these meters is very good, and as shown in data presented below for the current study, has advantages over single line and three line calibration expressions.

Prior to the workshop a large Ott Prop 1 meter was calibrated at five agencies. Johnstone (2002) discusses the comparative calibration results. Features of the five sets of calibration data for a large Ott Prop 1 are summarized in Table 2.

Agency 4 did not fit calibration lines to their data.

The principal current meters used within the USGS are the Price AA, and the Price Pygmy meters. The USGS have reported changing from using two linear segments to one straight line for their calibration expressions for these meters (Hubbard et al. 2001).

The variability of calibration points from the 5 agencies conforms to the ISO 748:2007 guideline for calibrations performed at several rating tanks.

Table 2: Summary of large Ott Prop 1 calibrations from five agencies.

Agency	$\sum$ pts	Sub-screw pts	Min V (m/s)	Max V (m/s)	Lines fitted
Agency 1	8	2	0.033	2.5	2
Agency 2	12	7	0.059	5.0	3
NIWA	25	16	0.056	1.99	1
Agency 3	21	15	0.112	2.09	1
Agency 4	24	15	0.069	2.42	-

3. Errors associated with current meter velocity measurements

The error in a velocity measurement has distinct random and systematic components.

It is likely that personnel carrying out current meter gaugings will be unaware of the magnitude of any systematic error due to a calibration equation misrepresenting the calibration data, and this error will not be identified or quantified with the gauging result. The systematic error cannot be reduced by repeat measurements or longer sampling times.

Random error associated with current meter performance is quantifiable and is independent of the fitted calibration equation. Random error can be reduced by repeat measurements. The uncertainty of a velocity result estimated as the average of a set of measurements will be smaller than that consisting of a single measurement. ISO 7066:1989 calls for accuracy tolerance limits for current meter ratings to be at the 95% confidence level, which is about 2 standard deviations from the mean.

The data shown in Table 3 are taken from ISO 748:2007, and are given as a guide to variability in meter performance based on experiments performed in several rating tanks. The uncertainty values shown are for individually rated meters (as is the NIWA practice).

Figure 7 below shows the variability of meter output observed for 5 successive calibration runs for a small Oss Prop 1 meter. The delta V values correspond to the difference between the rated velocities and the mean for the five runs expressed as a percentage. The absolute magnitude of the difference doesn't change greatly over the range of velocity. The higher percentage differences at low velocity reflect the low absolute values of the low velocities (the denominators in the % estimations).

Table 3: Variability expected from calibrations at several rating tanks

Velcotiy measured m/s	Uncertainties (% (95% confidence level))
0.03	± 20
0.10	± 5
0.15	± 2.5
0.25	± 2
0.50	± 1
Over 0.50	± 1

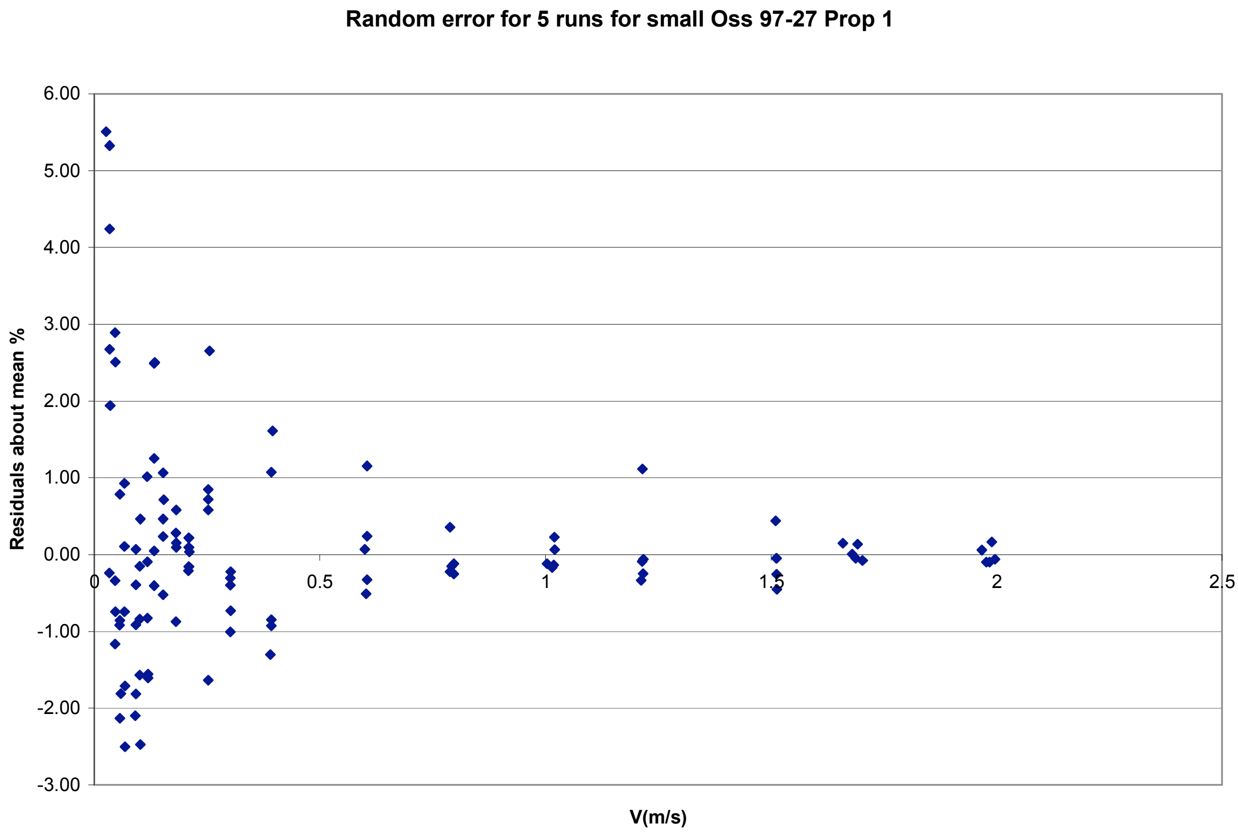

Figure 7: ΔV% values about mean for 5 successive calibrations of same meter

Sauer and Meyer (1992) discuss uncertainty associated with the Price AA and Pygmy meters. They found the random component of differences between different meters as being very small, and meter error to be large compared to other sources of error where velocity is slow.

4. An evaluation of the accuracy of NIWA calibration equations

ISO 748:2007 states that current meters should be used only within their calibrated range.

However, at the time of low flow measurements, there may be no alternative to using a meter well below a recommended range. Sensitivity curves highlight the change in meter output with changes in velocity, which is greater at low velocities than at high velocities. Consequently any systematic error due to imposing a straight line fit on calibration data will be much more significant at low velocities than at high velocities.

To allow for the variability exhibited in Figure 7, it is proposed that the targeted standard for agreement between velocity predicted by fitted lines, and the observed velocity for a calibration point should be within ± 5% for velocities less than 0.25m/s, and within ± 2% for velocities greater than or equal to 0.25m/s, throughout a recommended range.

To investigate the occurrence of calibration error for NIWA calibrations and its magnitude where it does occur, sample NIWA calibrations for several meter types have been examined for differences between the observed velocities for calibration points and the corresponding velocities predicted by the assigned calibration equations, within and beyond the recommended ranges.

Samples of low flow gaugings were checked to see whether velocity measurements below the recommended range do occur, and where they do occur, to assess the likely impact on accuracy of extrapolating a calibration line to estimate these velocities. The errors predicted apply to point velocity measurements, and contribute in part to the error in the mean velocity for a flow measurement.

The calibration errors referred to below are largely systematic in nature. They trend away to one side of the fitted straight lines, and could be greatly reduced by fitting alternative calibration models.

Each of the tables below includes a column that depicts the NIWA recommended range for the NIWA single straight line calibrations, for the particular meter type. Also depicted is the magnitude of a low point velocity measurement, taken from a sample of low flow gaugings, using the meter type.

The green region of the column shows the extent of the recommended range.

The red cell towards the top of the same column shows the magnitude of the sample low velocity measurement relative to the recommended range and the lowest calibration point.

There are two sets of tabulated reults. The first set shows the fit of normal NIWA calibration equations within their respective recommended ranges, and when extrapolated. The second set shows corresponding fit of multiple straight line models, and curvilinear or curvilinear and straight line models for some meters. Table 5, showing results for a single straight line fitted to all of an extended range of calibration points for a small Oss Prop1, is also included.

4.1. Small Ott Prop 1: NIWA single straight line, limited range

The NIWA recommended range for this meter type is: ***0.06 – 0.31m/s.***

A sample low velocity from a gauging: **0.025m/s.**

Table 4 lists the calibration points for an extended range calibration. However, the predicted velocities are calculated using the calibration equation derived from the normal NIWA limited range of points. The limited range of points is highlighted as the blue range in the Vobs column.

The limited range straight line calibration equation is seen to meet the target standard for agreement between the observed and predicted velocities within the recommended range, except for the highest point, but to exceed the standard when extrapolated up or down.

Table 4: Deviations of predicted V about the calibration line derived from the normal NIWA limited range of points

Small Ott 102859 Prop1					
Vobs (m/s)	Rotns n (rev/s)	Vpred (m/s)	Vobs-Vpred (m/s)	ΔV%	V (m/s)
					0.025
0.034	0.286	0.037	-0.004	-11.57	
0.050	0.529	0.052	-0.003	-5.48	
0.058	0.658	0.060	-0.003	-4.63	
0.067	0.787	0.068	-0.001	-1.62	0.06
0.092	1.153	0.091	0.001	0.74	
0.102	1.311	0.101	0.001	1.18	
0.119	1.575	0.117	0.002	1.57	
0.133	1.800	0.131	0.003	1.90	
0.153	2.115	0.150	0.003	1.78	
0.181	2.574	0.178	0.003	1.61	
0.210	3.029	0.206	0.003	1.64	
0.253	3.723	0.249	0.005	1.78	
0.307	4.811	0.316	-0.009	-2.99	0.31
0.392	6.324	0.409	-0.017	-4.29	
0.603	10.382	0.658	-0.056	-9.24	
0.795	14.091	0.886	-0.091	-11.46	
1.019	18.332	1.147	-0.128	-12.55	
1.216	22.326	1.393	-0.177	-14.55	
1.510	28.022	1.743	-0.233	-15.43	
1.689	31.453	1.954	-0.265	-15.68	
1.983	37.130	2.303	-0.321	-16.18	

4.2. Small Oss Prop 1: A straight line fitted to data for an extended range

The NIWA recommended range for this meter type is: **0.06 – 0.31m/s.**

A sample low velocity from a gauging: **0.025m/s.**

Data presented in Table 5 show the fit of a single straight line derived using all the extended range calibration points

Table 5: Deviations of predicted V about a single straight line fitted to the extended range of calibration points

Small Oss 97-27 Prop 1				
Calibration points				
V	Rotns n	vobs-vpred	% diff.	V
(m/s)	(rev/s)			m/s
				0.025
0.0327	0.2687	-0.016	-48.1	
0.046	0.4727	-0.013	-29.2	
0.056	0.6231	-0.012	-20.6	
0.0672	0.7998	-0.010	-14.7	0.06
0.0916	1.185	-0.006	-6.9	
0.1005	1.3293	-0.005	-5.1	
0.1179	1.6268	-0.004	-3.2	
0.1322	1.8573	-0.002	-1.5	
0.1524	2.1923	0.000	0.1	
0.1809	2.6651	0.003	1.7	
0.2089	3.1702	0.004	1.9	
0.2527	3.8462	0.011	4.4	
0.3008	4.7555	0.010	3.4	0.31
0.3898	6.3322	0.014	3.6	
0.6046	10.3141	0.014	2.3	
0.7972	13.844	0.016	2.0	
1.0037	17.7094	0.014	1.4	
1.2167	21.7947	0.007	0.5	
1.5135	27.3839	0.002	0.1	
1.6919	30.8912	-0.009	-0.5	
1.9842	36.492	-0.019	-1.0	

These data show that the single straight line representation of the calibration will introduce large errors for velocities below 0.1m/s. The magnitude of errors can be expected to increase with extrapolation down towards 0.025m/s.

4.3. Large Ott C31 P2: NIWA single straight line, limited range

The NIWA recommended range for this meter type is: 0.1 – 6.0m/s.

A sample low velocity from a gauging: 0.01m/s.

The quality of the normal NIWA straight line fit to calibration data for a large Ott Prop2 is shown in Table 6.

Table 6: Deviations of predicted V from the normal NIWA straight calibration line

Large Ott Prop 2					
		V = 0.4804n + 0.008			
Test 2 cal data		R2 = 1			
		Linear Regression	1 straight line		
Vobs	n	Vpred	Vobs-Vpred	delta V%	V
					0.01
0.0682	0.0996	0.056	0.012	18.11	
0.08	0.1404	0.075	0.005	5.69	
0.0884	0.1583	0.084	0.004	4.92	
0.1009	0.1888	0.099	0.002	2.18	0.100
0.1205	0.2323	0.120	0.001	0.75	
0.1315	0.2556	0.131	0.001	0.54	
0.1518	0.2995	0.152	0.000	-0.05	
0.1806	0.3633	0.183	-0.002	-1.07	
0.2081	0.4228	0.211	-0.003	-1.45	
0.2546	0.5183	0.257	-0.002	-0.94	
0.3055	0.6263	0.309	-0.003	-1.10	
0.4062	0.8401	0.412	-0.005	-1.33	
0.6007	1.2395	0.603	-0.003	-0.46	
0.8237	1.703	0.826	-0.002	-0.29	
1.0032	2.0772	1.006	-0.003	-0.27	
1.1949	2.4733	1.196	-0.001	-0.11	
1.3876	2.8753	1.389	-0.002	-0.12	
1.6561	3.4337	1.658	-0.001	-0.09	
1.8022	3.7367	1.803	-0.001	-0.05	
2.0034	4.1617	2.007	-0.004	-0.19	
2.1928	4.5566	2.197	-0.004	-0.19	
2.3733	4.9314	2.377	-0.004	-0.16	
2.6872	5.5639	2.681	0.006	0.23	
2.996	6.2038	2.988	0.008	0.26	
					6

The single line calibration representation is seen to underestimate V significantly at the low end, and the magnitude of the error can be expected to increase when the equation is extrapolated down towards 0.01m/s.

4.4. Price AA Meter: NIWA single straight line, limited range

The NIWA recommended range for this meter type is: **0.08 – 3.66m/s.**

A sample low velocity from a gauging: **0.02m/s.**

The normal NIWA single straight line is used to estimate V_{pred} for the calibration in Table 7.

Table 7: Deviations of predicted V using the normal NIWA straight calibration line

Price AA 384 RS					
		V=0.677*n+0.007			
		Linear regression 1 straight line			
Vobs	**n**	**Vpred**	**Vobs-Vpred**	**deltaV%**	**V**
					0.02
0.078	0.106	0.079	-0.001	-1.45	0.080
0.105	0.147	0.107	-0.002	-1.45	
0.155	0.219	0.155	0.000	0.13	
0.233	0.334	0.233	-0.001	-0.25	
0.344	0.497	0.343	0.001	0.19	
0.495	0.723	0.497	-0.002	-0.31	
0.670	0.978	0.669	0.001	0.13	
0.929	1.371	0.935	-0.006	-0.68	
1.214	1.775	1.208	0.006	0.47	
1.531	2.247	1.528	0.003	0.17	
1.840	2.710	1.842	-0.002	-0.09	
2.190	3.237	2.198	-0.008	-0.37	
2.475	3.643	2.473	0.002	0.07	
					3.66

The calibration line error for the range 0.07 – 0.1m/s is seen be small, and meets the target standard, although extrapolation down to 0.02m/s can be expected to introduce a larger error.

4.5. Price Pygmy meter: NIWA single straight line, limited range

The NIWA recommended range for this meter type is: 0.08 – 2.00 m/s.

A sample low velocity from a gauging: 0.023 m/s.

The normal NIWA single straight line is used to represent the calibration in Table 8.

Table 8: Deviations of predicted V using the normal NIWA straight calibration line

Pygmy 671691RS

Single straight line: V = 0.302n + 0.020

Vobs	n	Vpred	Vobs-Vpred	DeltaV%	V
					0.023
0.062	0.144	0.064	-0.002	-3.01	
0.092	0.251	0.096	-0.004	-4.70	0.08
0.121	0.338	0.122	-0.001	-1.13	
0.182	0.538	0.182	-0.001	-0.40	
0.238	0.722	0.238	0.000	-0.14	
0.302	0.938	0.303	-0.001	-0.30	
0.428	1.342	0.425	0.003	0.63	
0.549	1.731	0.543	0.006	1.12	
0.718	2.265	0.704	0.013	1.88	
0.927	3.008	0.928	-0.001	-0.11	
1.426	4.645	1.423	0.003	0.24	
1.983	6.506	1.985	-0.002	-0.09	2.00

The calibration error at the lowest calibration point exceeds the target standard

Extrapolating the single straight line Pygmy calibration down to 0.02m/s is likely to incur a larger error.

4.6. Small Oss 97-27, Prop 1: Three straight line segments fitted to extended range calibration points

The NIWA recommended range for this meter types is: 0.06 – 0.31m/s.

A sample low velocity from a gauging: 0.025m/s.

Table 9 lists the extended range calibration points from an Agency 2 calibration of this meter. Also included are the NIWA points for a calibration of the same meter that extend down below the Agency 2 calibration points. Although Agency 2 uses three linear equations for the extended calibration range, their calibration points don't extend as low as the lowest NIWA calibration point. The three Agency 2 calibration lines are used to predict velocities for the respective Agency 2 segments, and the lowest Agency 2 line is also used to predict velocities for the NIWA points that extend below the Agency 2 calibration.

Table 9: Combined Agency 2 calibration points, and lower NIWA points. Deviations of predicted V calculated using the 3 Agency 2 lines fitted to the Agency 2 points

Small Oss 97-27 Prop1

Using lowest Agency 2 equation to extrapolate down to lowest NIWA points

	V	n	Vpred	delta V	deltaV%		V
							0.02
NIWA	0.033	0.269	0.037	-0.005	-14.15	From	
data	0.046	0.473	0.049	-0.003	-7.27	Agency 2	
	0.056	0.623	0.058	-0.002	-3.93	line1	
Agency 2	0.059	0.643	0.059	0.000	-0.30	Agency 2	0.06
data	0.079	0.973	0.079	0.000	0.47	line 1	
	0.099	1.321	0.099	0.000	-0.19		
	0.119	1.677	0.120	-0.001	-0.88	Agency 2	
	0.150	2.190	0.149	0.001	0.46	line 2	
	0.182	2.733	0.180	0.001	0.80		
	0.400	6.613	0.400	-0.001	-0.17		
	0.802	14.229	0.808	-0.006	-0.70	Agency 2	0.31
	1.205	21.578	1.199	0.006	0.48	line 3	
	1.605	29.042	1.597	0.008	0.48		
	2.005	36.876	2.015	-0.010	-0.49		

The Agency 2 lines are seen to provide a good fit to the calibration data for velocities across the range of the Agency 2 calibration. However the results above show that significant errors are incurred when the lowest Agency 2 is line extrapolated downwards. The calibration error at 0.025m/s is expected to higher. To express this set of calibration data, including the NIWA points as a set of linear segments meeting the target standard, at least one additional straight line segment would be needed.

4.7. Small Oss Prop 1: A Beta function fitted to data for an extended range

The NIWA recommended range for this meter type is: 0.06 – 0.31m/s.

A sample low velocity from a gauging: 0.025m/s.

Data presented in Table 10 show the fit of a Beta function line derived for the same extended range calibration points listed in Table 5.

Table 10: Deviations of predicted V about a Beta function line fitted to the extended range of calibration points

Small Oss 97-27 Prop 1				
Calibration points		**Beta function**		
v	Rotns n	vobs-vpred	% diff.	V
(m/s)	(rev/s)			m/s
				0.025
0.033	0.269	-0.004	-11.1	
0.046	0.473	0.003	7.5	
0.056	0.623	0.000	0.5	
0.067	0.800	0.000	-0.1	0.06
0.092	1.185	0.002	2.2	
0.101	1.329	-0.001	-1.3	
0.118	1.627	-0.001	-1.1	
0.132	1.857	-0.002	-1.3	
0.152	2.192	-0.001	-0.7	
0.181	2.665	0.000	0.0	
0.209	3.170	-0.002	-0.9	
0.253	3.846	0.001	0.4	
0.301	4.756	-0.003	-0.9	0.31
0.390	6.332	0.000	-0.1	
0.605	10.314	0.001	0.2	
0.797	13.844	0.006	0.7	
1.004	17.709	0.006	0.6	
1.217	21.795	0.003	0.2	
1.514	27.384	0.004	0.2	
1.692	30.891	-0.003	-0.2	
1.984	36.492	-0.006	-0.3	

The Beta function fit for the small Oss Prop 1 calibration data is significantly better than the straight line fit shown in Table 5, although the fit for the lowest two points suggests that it is not suitable for downward extrapolation.

4.8. Large Ott C31 P2: Straight line, zero intercept fit to Screw range, third order polynomial fit to sub-screw range

The NIWA recommended range for this meter type is: 0.1 – 6.0m/s.

A sample low velocity from a gauging: 0.01m/s.

The set of calibration points listed in Table 11 is the same set as used for the test summarized in Table 6, however this time a straight line with a zero intercept is used to represent the screw state, and a third order polynomial function is used to represent the sub screw state.

Table 11: Deviations of predicted V calculated using a straight line (zero intercept) for the screw range, and a 3rd order polynomial for the sub screw range

Ref Large Ott Prop 2
Test 2 cal data

Vobs	n	Vpred	Vobs-Vpred	deltaV%	V	
		1 cubic line & 1 straight line with 0 intercept				
					0.01	
0.068	0.100	0.064	0.004	6.54		
0.080	0.140	0.081	-0.001	-1.73		
0.088	0.158	0.089	-0.001	-0.89		
0.101	0.189	0.103	-0.002	-1.65	0.10	
0.121	0.232	0.122	-0.001	-1.09		
0.132	0.256	0.132	-0.001	-0.54		$y = -0.0199x^3 + 0.0651x^2 + 0.4178x + 0.0215$
0.152	0.300	0.152	0.000	-0.09		$R^2 = 0.9999$
0.181	0.363	0.181	0.000	-0.18		
0.208	0.423	0.208	0.000	-0.09		
0.255	0.518	0.253	0.002	0.72		
0.306	0.626	0.304	0.002	0.55		
0.406	0.840	0.407	0.000	-0.11		
0.601	1.240	0.601	-0.001	-0.13		
0.824	1.703	0.821	0.002	0.28		
1.003	2.077	1.002	0.001	0.14		
1.195	2.473	1.193	0.002	0.17		
1.388	2.875	1.387	0.001	0.06		
1.656	3.434	1.656	0.000	0.00		$y = 0.4823x$
1.802	3.737	1.802	0.000	0.00		$R^2 = 1$
2.003	4.162	2.007	-0.004	-0.19		
2.193	4.557	2.198	-0.005	-0.22		
2.373	4.931	2.378	-0.005	-0.22		
2.687	5.564	2.683	0.004	0.14		
2.996	6.204	2.992	0.004	0.13		

The straight line, zero intercept equation meets the target standard for the screw range points. The third order polynomial expression for the lower end calibration shows a better fit than the single straight line (Table 6), although the error for the bottom point in this instance suggests that the polynomial expression may not be suitable for extrapolation downwards.

5. Conclusions

The low end of the NIWA recommended range for all meters is generally ≥ 0.06 m/s.

Low flow measurements in New Zealand sometimes require velocity measurements as low as 0.01 m/s.

The NIWA rating car can provide calibration points as low as 0.025 m/s, possibly lower.

On the basis of the tests carried out in this study, the magnitude of systematic calibration error associated with NIWA calibrations can be summarized as follows:

Meter Type	Velocity	Calibration Error
Small Ott/Oss Prop 1;	within recommended range:	small, within the target standard
	below recommended range	large, outside the target standard
	above recommended range	large, outside the target standard
Large Ott Prop 2	within recommended range	small, within the target standard
	below recommended range	large, outside the target standard
	above recommended range	very small at top calibration point
Price AA	within recommended range	small, within the target standard
	below recommended range	unclear, probably large
	above recommended range	very small at top calibration point
Pygmy	within recommended range	within the target standard
	below recommended range	unclear, probably large
	above recommended range	very small at top calibration point

The systematic errors associated with extrapolating the NIWA calibration equations to velocity values down towards the lowest values gauged will be large for all meter types, because of the marked change in meter output at very low velocity.

Sensitivity curves show that for all meter types, the rate of change of meter output increases with decreasing velocity, indicating that the rate of change of slope in the V versus n calibration line is greatest at very low velocity. Calibration points are seen to diverge from a straight calibration line at the low velocity end. A curvilinear function representing the low end of a calibration line would allow more accurate prediction of low velocity than a straight line.

The results for the small Oss Prop 1 calibration (Table 9) show that for this meter type three straight lines cannot meet the target standard for the velocity range 0.025 – 2.0 m/s.

A minimum of four straight lines (requiring 8 parameters – 4 slopes and 4 constants) would be required to represent the calibration as a set of linear segments covering this velocity range.

For meters that attain the stable screw state, it is expected that a meter output in terms of revs/m will remain stable, i.e. conform to a straight line with a zero intercept, for velocities above 2 m/s. The 2002 Agency 2 calibration data for the large Ott Prop 1 meter used for the inter-agency comparisons, confirms that the stable screw state persisted up to the highest calibration velocity of 5.0 m/s.

Use of curvilinear functions for all or part of calibration ranges would represent a departure from the detail of the ISO 3455 procedure for deriving the calibration equations. Prior to the advent of PCs and gauging computers the expression of calibration data was constrained to linear segments. Nowadays, in cases where non-linear expressions allow significantly more accurate application of a calibration, it is sensible to use non-linear equations.

6. Recommendations arising from this review are:

That calibration points are extended down to 0.025 m/s, or lower if possible.

That for each meter type, functions are researched that are characteristic of meter behaviour for the full velocity range of the rating car. Ideally the selected functions will allow reliable extrapolation where that is required. This research will include further evaluation of the beta function, especially as a calibration function for the small Ott and Oss meters, Prop1. It will be necessary to carry out a number of repeat calibrations to allow the mean behaviour of a meter type to be determined across the extended range. Ideally, a greater number of repeat calibration points at selected velocity values will be collected to allow estimation of the uncertainty of calculated velocity at 95% confidence limits, across the velocity range.

That for meters that attain the screw state, the calibration for that range be expressed as a straight line with a zero intercept.

That the minimum number of calibration points required to adequately characterize the extended velocity range for each meter type be determined, to contribute to a specification for calibration of the respective meter types. It is likely that relatively few points are necessary to define the screw state for meters that attain that state, and that more points are required to define the low velocity range.

That a set of rules and tests be defined for the creation of an Excel macro to automate fitting of new calibration equations.

That necessary modifications to Gauge software are specified, to allow new calibration equations to be implemented.

That calibration error, and expected random calibration error across the calibration range (as 95% confidence limits) are displayed on calibration certificates.

That for meters used for flood gauging, confirmation of a stable output at high velocity in terms of revs/meter, is sought from calibration agencies whose rating cars can travel at speeds up to 5 m/s or greater.

7. Acknowledgement

This review was funded by an Envirolink small advice grant.

8. References

DeZeeuw, C.; Bil, C. (1975). Repair and maintenance manual of the Price current meter. Canada Centre for Inland Water, National Calibration Service Burlington, Ontario: p 5.

Engel, P. (1999). Current Meter Calibration Strategy. *Journal of Hydraulic Engineering / December 1999*.

Fulford, Janice M.; Thibodeaux, K.G.; Kaehrle, William R. (1993). Repeatability and Oblique Flow Response Characteristics of Current Meters. *Journal of Hydraulic Engineering. Proceedings of the 1993 conference*.

Herschy, R.W. (1985). Streamflow Measurement, Elsevier Applied Science Publishers, New York. 453 p.

Hubbard, E.F.; Schwarz, G.E.; Thibodeaux, K.G.; Turcios, Lisa M. (2001). Price Current- meter Standard Rating Development ByThe U.S. Geological Survey. *Journal of Hydraulic Engineering / April 2001*.

Hubbard, E.F.; Thibodeaux, K.G.; Duong, Mai N. (1999). Quality Assurance of U.S. Geological Survey Stream Current Meters: The Meter Exchange Program 1988-98: U.S. Geological Survey, Open-File Report 99-221.

ISO 748:2007. Measurement of liquid flow in open channels – Velocity-area methods.

ISO 2537:2007. Liquid flow measurement in open channels – Rotating element current-meters.

ISO 3455:2007. Liquid flow measurement in open channels – Calibration of rotating-element current-meters in straight open tanks.

ISO 7066-1:1997. Assessment of uncertainty in the calibration and use of flow measurement devices – Part 1: Linear calibration devices.

Johnson, R.L. (1966). Laboratory Determination of Current Meter Performance. Technical Report No. 843-1. US Department of Commerce.

Johnstone, D.E. (2002). An analysis of the five agencies comparative calibrations of an Ott C31 prop 1 meter.

Johnstone, D.E. (2002). Current Meter Behaviour and Calibration Equations. NIWA Report.

McKerchar, A.M.; Johnstone, D.E (2002). Calibration of Current Meters. Poster Paper, NZ Hydrological Society Symposium.

Sauer, V.B.; Meyer, R.W. (1992). Determination of errors in individual discharge measurements: *US Geological Survey, Open-File Report*: 92–144.

Thibodeaux, K.G. (1994). Review of Literature on the Testing of Point-Velocity Current Meters: *U.S. Geological Survey Open-File Report*: 94-123.