New extended-range calibration equations for Small Ott and Pygmy current meters

NIWA Client Report: CHC2009-057
May 2009

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Tasman District Council

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Contents

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1. Introduction

The NIWA Client Report CHC2008-144 entitled "Current Meter Calibration Equations: A review of the adequacy of limited range equations for New Zealand requirements" backgrounds and describes the widespread practice of expressing current meter calibrations as one or several straight line segments. Calibrations carried out at NIWA's tow tank facility are expressed as a single straight line applicable to a recommended limited velocity range. A review of low flow gaugings showed that gauging personnel often have no choice but to attempt to measure velocities well below recommended ranges, and a need was identified for calibration expressions applicable to a wider velocity ranges, especially to very low velocities. The suitability of NIWA calibration equations for extrapolation outside the recommended range was analysed for a range of meter types. Extrapolation downwards in particular, was shown to introduce a pronounced systematic misrepresentation of the calibration data. A conclusion of the study reported in CHC2008-144 was that a curvilinear function would allow more accurate prediction of velocities than a straight line, particularly low velocities. The report recommends researching functions that are characteristic of meter behaviour over extended velocity ranges, and that uncertainty is estimated across the calibration ranges and displayed on calibration certificates. The results of work following on from the study reported in CHC2008-144 are presented here. Funding constraints restricted research on new calibration equations to two meter types. The small Ott Prop 1, and reed switch (RS) Pygmy meters were considered to have the highest priority because of a focus on low velocity measurement.

For comparison with results for new equations investigated in this second phase of the project, some results from CHC2008-144 for the small Ott Prop 1 and RS Pygmy type meters are reproduced here. These are tables showing the fit of the limited range NIWA equations, particularly when extrapolated beyond their recommended ranges. Table 1 shows the results for a small Ott Prop1 type meter calibration and Table 2 those for a RS Pygmy meter calibration. These tables are reproductions of Tables 4 and 8 respectively from the earlier report.

Each of the tables includes a column that depicts the NIWA recommended range for the NIWA single straight line calibration, for the particular meter type. Also depicted is the magnitude of a low point velocity measurement, taken from a sample of low flow gaugings, using the meter type.

The green region of the column shows the extent of the recommended range.

The red cell towards the top of the same column shows the magnitude of a sample low velocity measurement from a gauging using the meter type, relative to the recommended range and the lowest calibration point.

Table 1 and Table 2 lists the calibration points for an extended range calibration. However, the predicted velocities are calculated using the calibration equation derived from the normal NIWA limited range of points. The limited range of points is highlighted as the blue range in the Vobs column.

Table 1: Deviations of predicted V about the calibration line derived from the normal NIWA limited range of points for a small Ott Prop 1 meter

Small Ott 102859 Prop1					
Vobs (m/s)	Rotns n (rev/s)	Vpred (m/s)	Vobs-Vpred (m/s)	ΔV%	V (m/s)
					0.025
0.034	0.286	0.037	-0.004	-11.57	
0.050	0.529	0.052	-0.003	-5.48	
0.058	0.658	0.060	-0.003	-4.63	
0.067	0.787	0.068	-0.001	-1.62	0.06
0.092	1.153	0.091	0.001	0.74	
0.102	1.311	0.101	0.001	1.18	
0.119	1.575	0.117	0.002	1.57	
0.133	1.800	0.131	0.003	1.90	
0.153	2.115	0.150	0.003	1.78	
0.181	2.574	0.178	0.003	1.61	
0.210	3.029	0.206	0.003	1.64	
0.253	3.723	0.249	0.005	1.78	
0.307	4.811	0.316	-0.009	-2.99	0.31
0.392	6.324	0.409	-0.017	-4.29	
0.603	10.382	0.658	-0.056	-9.24	
0.795	14.091	0.886	-0.091	-11.46	
1.019	18.332	1.147	-0.128	-12.55	
1.216	22.326	1.393	-0.177	-14.55	
1.510	28.022	1.743	-0.233	-15.43	
1.689	31.453	1.954	-0.265	-15.68	
1.983	37.130	2.303	-0.321	-16.18	

It was proposed that targeted specification for agreement between velocity predicted by fitted lines, and the observed velocity for a calibration point should be: within ± 5% for velocities less than 0.25m/s, and within ± 2% for velocities greater than or equal to 0.25m/s, throughout a recommended range.

The limited range straight line calibration equation is seen to meet the target specification for agreement between the observed and predicted velocities within the recommended range, except for the highest point, but does not meet the standard when extrapolated up or down.

Table 2: Deviations of predicted V about the calibration line derived from the normal NIWA limited range of points for a reed switch Pygmy meter

Pygmy 671691RS

Single straight line: V = 0.302n + 0.020

Vobs	n	Vpred	Vobs-Vpred	DeltaV%	V
					0.023
0.062	0.144	0.064	-0.002	-3.01	
0.092	0.251	0.096	-0.004	-4.70	0.08
0.121	0.338	0.122	-0.001	-1.13	
0.182	0.538	0.182	-0.001	-0.40	
0.238	0.722	0.238	0.000	-0.14	
0.302	0.938	0.303	-0.001	-0.30	
0.428	1.342	0.425	0.003	0.63	
0.549	1.731	0.543	0.006	1.12	
0.718	2.265	0.704	0.013	1.88	
0.927	3.008	0.928	-0.001	-0.11	
1.426	4.645	1.423	0.003	0.24	
1.983	6.506	1.985	-0.002	-0.09	2.00

The calibration error at the lowest calibration point does not meet the target standard.

Extrapolating the single straight line Pygmy calibration down to 0.023m/s is likely to incur a larger error.

2. Evaluation of non-linear equations to represent calibrations for the small Ott Prop 1 type and Reed Switch Pygmy type meters

The objective of this stage of the project is to meet the requirement for a calibration curve for both the small Ott Prop 1 type and Reed Switch Pygmy meters which in each case:

1. Can represent the "true" relationship between rotation speed and water velocity with bias of less than 2% through a wide range of rotation speeds.

2. Has parameters that can be reliably estimated from a suitably-screened individual current meter calibration data sets.

3. Has a mathematical form that captures the "character" of the meter.

The data sets used for evaluating equations consisted of 10 extended-range calibrations collected for a meter of each type. Each small Ott Prop 1 calibration is comprised of 20 points, with the exception of calibration 1 (19 points only). Each Pygmy calibration is comprised of 21 points. The velocity of the NIWA car is set using an analog dial and actual car velocities for each repetition in a set of 10, for a particular nominal velocity, varies slightly from the nominal value.

The velocity for the lowest velocity point in each of the 10 calibrations was the lowest velocity for which the rotor would continue to rotate, without stopping. This velocity was determined by starting the car and increasing the velocity until the rotor began to rotate, then reducing the car velocity till the minimum speed capable of sustaining rotation was found. The small Ott Prop 1 meter was found to require a minimum velocity of 0.023m/s to sustain rotation. The reed switch Pygmy was found to require a minimum velocity of 0.04m/s to sustain rotation.

2.1. Small Ott Prop 1 type meters

Extended range calibration data were for 10 calibration runs for the small Ott Prop1 meter: C2-170993, propeller 1-171647. The nominal velocity values for the 20 points for each calibration range from 0.025 – 2.1 m/s.

The data from all 10 runs are shown in Figure 1. The most obvious feature of the data is the approximately linear relationship between rotation rate and velocity. The straight line on the plot below (data from all 10 runs) and the theoretical concept of the "screw state" of a meter, both suggest that the calibration curve should be the sum of a straight line and a deviation from that line.

Functions initially investigated for the small Ott Prop 1 type meter were a Beta function, Engel's[1]function, a cubic (third order polynomial) function and a sequence of three spliced quadratics.

[1] Engel, P. (1999). Current Meter Calibration Strategy. Journal of Hydraulic Engineering 125(12): 1306-1308.

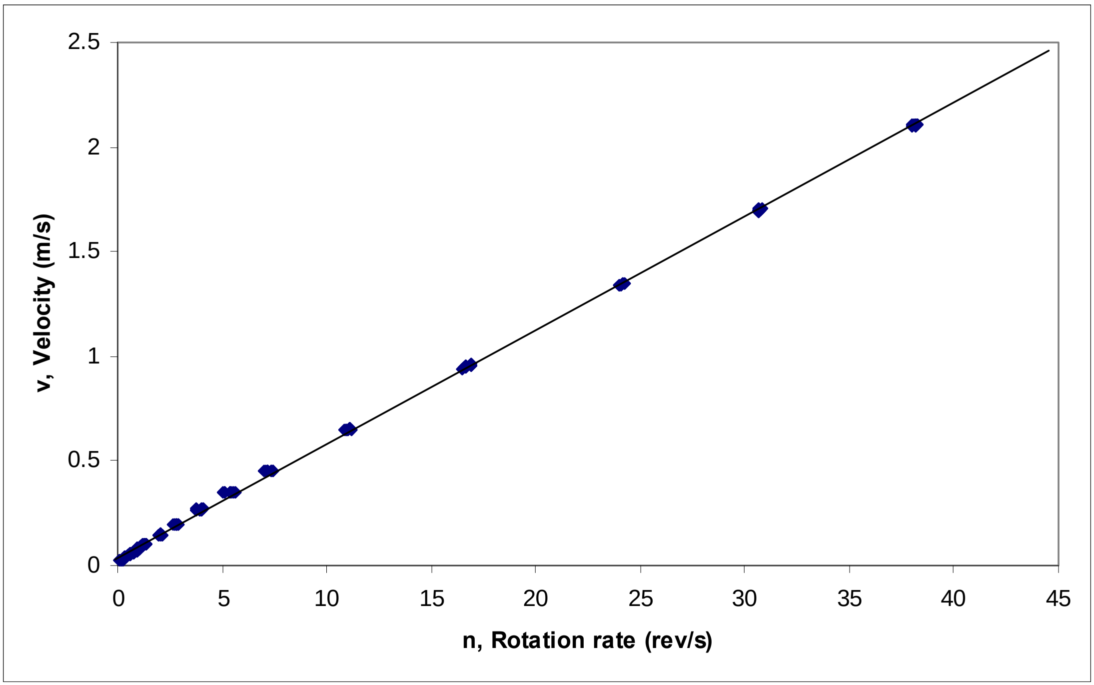

Figure 1: **Data from the 10 small Ott Prop1 runs**

The Beta function has the form:

$$v = kn + dn^a(c - n)^b$$

where k is the propeller pitch, and there are four parameters $(a,b,c$ & $d)$ to be estimated;

The equation developed by Engel for Price meters has the form:

$$v = An + B\exp(-Kn)$$

where A, B and K are coefficients to be determined.

The cubic form is:

$$v = an^3 + b\,n^2 + cn + d$$

where a,b,c and d are parameters to be estimated.

Each of the three spliced quadratics has the form:

$$v = a\,n^2 + bn + c$$

where a, b, and c are parameters to be estimated for each equation. Nine parameters are required to represent the calibration.

At low velocities, the region of particular interest, the Beta, Engel and cubic functions did not fit the calibration data well. In each case there was significant systematic error. The spliced quadratic equations were able to fit the data well, without evidence of systematic mis-representation of the calibration data. However this last model was not favoured because of the requirement for 9 parameters.

2.1.1. The Woods equation

A fifth form of function was developed by Ross Woods using the rationale described here:

Figure 2 and Figure 3 show plots of the deviation of velocity from a straight-line relationship; Figure 2 shows the full range of data and Figure 3 shows only the lowest rotation rates. The square plotting symbols labelled “Vbar-k*n” show the mean values across all 10 runs (except the slowest speed on run 1 which is missing): this is the best estimate of the true meter characteristic. These plots help identify the functional forms that might be used for the deviation component of the calibration curve. We have assumed k=0.5505 m/rev.

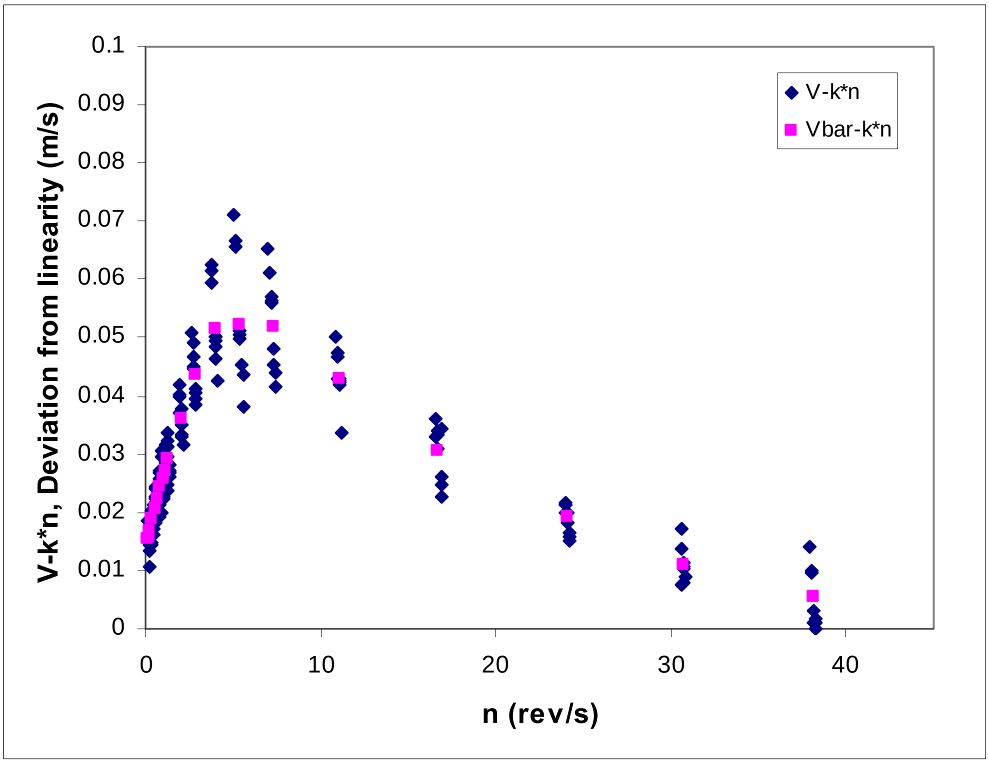

Figure 2: Velocity deviation from linearity (V-kn) plotted against rotation rate, for small Ott prop 1.

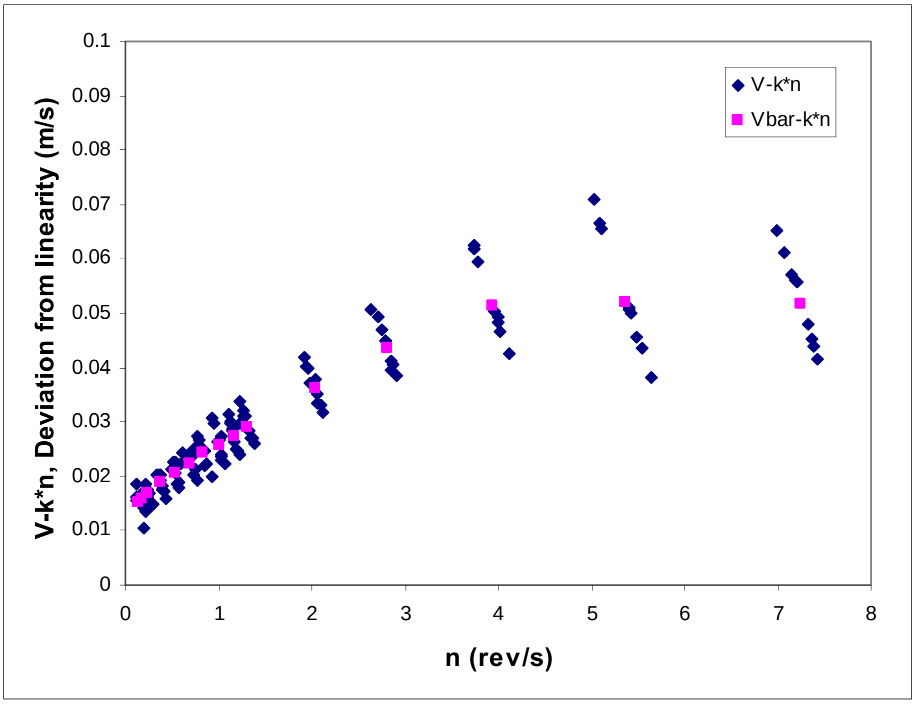

Figure 3: Velocity deviation from linearity (V-kn) plotted against rotation rate (low speeds), for small Ott prop 1.

Key features of Figure 2 and Figure 3 are:

1. V-k*n is apparently non-zero as n decreases to 0

2. V-k*n increases linearly (or perhaps quadratically) as n increases from 0

3. V-k*n reaches a maximum somewhere between n=3 and n=8

4. V-k*n decreases for n>8

5. If k is chosen to model the screw state of the meter (so that V=k*n for very large n), then V-k*n declines asymptotically towards 0 as n increases

If we are required to mimic the increase and decrease in the deviation from linearity, then a function with more than three parameters will be needed – functions such as Engel's will not be adequate

The Woods equation has the form:

$$V = k\,n + v_0\,exp(-n/n_0) + a\,(n/n_0)^p\,exp(-n/n_0)$$

(i) (ii) (iii)

There are 5 parameters: ***k, v_0, a, n_0, p***. If a=0, this equation is the same as Engel's (his A = our k, his B = our v_0, his k = our $1/n_0$).

The curve has 3 parts: (i) is the screw state, that is, the asymptotically linear response of the meter at high velocities; (ii) produces the apparent non-zero velocity for n approaching 0; (iii) is the remaining deviation from the screw state and non-zero velocity that is not otherwise explained.

Some of the calibration curve parameters have an interpretation:

- k is the apparent pitch of the meter (m/revolution),
- v_0 is the apparent minimum velocity of the meter (m/s)
- a controls the size of the maximum deviation from linearity (m/s) (maximum deviation $\approx v_0 + a\, p^p \exp(-p)$)
- n_0 controls the rotation rate at which the maximum deviation from linearity occurs (revs/s) ($n_{max} \approx pn_0$)
- p controls the shape of the deviation curve (especially important for low n)

The parameter estimation technique used to estimate these 5 model parameters was chosen to minimize the percentage bias at all velocities.

2.1.2. Evaluation of Woods equation for the small Ott Prop 1 type meters

Optimal parameter values obtained for each of the 10 calibration runs are shown in Table 3. The optimal values listed in the column "Mean" were obtained by fitting to an average set of n-v data formed by taking the average n and v across all 10 runs, for each speed number. The v-n data for the mean case are the best available estimate of the true meter response.

Table 3: Optimised parameter values for each calibration run (Small Ott C2-170993 10 runs)

	run1	run2	run3	run4	run5	run6	run7	run8	run9	run10	Mean
k	0.055	0.055	0.055	0.056	0.055	0.055	0.055	0.056	0.056	0.055	0.055
v0	0.018	0.013	0.015	0.014	-0.08	0.015	0.015	0.015	0.014	0.016	0.014
n0	5.764	7.250	6.457	4.594	42.85	5.671	5.197	3.410	4.293	8.238	6.730
a	0.125	0.160	0.159	0.124	0.148	0.150	0.119	0.096	0.109	0.103	0.129
p	1.250	1.007	1.136	1.125	0.099	1.225	1.355	1.607	1.327	1.052	1.077

The parameter values for run5 are clearly anomalous, and point to a significant outlier in this data set. The three highlighted values are all well outside the expected range of values for this meter (**k**: 0.054-0.056; **v0**: 0.011 - 0.018; **n0**: 3-8; **a**: .1-.16; **p**: 1-1.6)

The discrepancies between measured and modelled velocity for each run are shown in Table 4. This table can be compared to Table 1, which highlights the limitations of the current limited range NIWA calibration equations for this meter type. No consistent evidence of bias is seen across the extended velocity range in the results for the Woods equation.

Table 4: Discrepancies between modelled and measured velocities for each run (Small Ott C2-170993 10 runs)

Mean v	run1 Delta V%	run2 Delta V%	run3 Delta V%	run4 Delta V%	run5 Delta V%	run6 Delta V%	run7 Delta V%	run8 Delta V%	run9 Delta V%	run10 Delta V%	Mean Delta V%
0.023		0.57	-0.45	0.64	3.82	0.05	-0.85	-3.02	-1.73	-2.64	0.44
0.026	-1.02	0.84	1.90	-0.75	-4.73	0.87	0.91	1.87	2.64	5.53	0.42
0.030	0.77	-0.35	-0.69	0.45	-0.07	0.71	0.61	2.79	-0.41	-0.50	0.05
0.040	0.06	-1.92	-1.17	-1.02	-2.10	-1.07	0.18	0.24	-0.16	-2.38	-0.97
0.050	0.19	0.80	-1.53	-0.05	-0.74	-1.39	0.50	-1.64	0.20	-0.56	-0.46
0.061	0.24	-0.29	-1.52	1.32	-0.91	-0.28	-0.70	-0.53	0.74	2.10	0.13
0.070	0.04	-0.67	-0.04	0.27	-0.11	0.09	-0.60	0.74	0.37	-5.16	-0.43
0.081	-0.27	-1.43	2.49	-0.45	4.38	1.08	0.19	-0.41	0.34	1.80	0.85
0.091	-0.24	0.57	0.85	1.79	3.24	0.61	0.31	-0.21	-0.72	1.92	0.87
0.101	-0.56	2.01	1.32	-0.81	1.56	0.22	0.04	-0.64	-0.66	1.98	0.50
0.148	-0.64	1.87	0.23	-0.74	-0.62	1.67	0.66	0.76	-0.02	0.16	0.26
0.198	-1.14	1.61	-0.68	-2.15	-2.63	1.33	0.27	0.32	0.03	-1.76	-0.61
0.269	2.64	-2.16	-1.48	0.75	-4.66	-2.65	-1.05	-0.62	-0.58	-1.16	-1.16
0.348	0.76	-2.73	-1.40	0.85	0.47	-1.88	0.00	0.35	1.54	1.06	-0.02
0.451	-1.57	-0.43	1.09	0.77	-3.31	1.08	0.83	0.32	0.63	0.05	0.19
0.650	-0.49	1.37	1.50	-0.94	-0.94	1.43	0.18	-0.59	-0.99	0.19	0.43
0.951	0.47	1.51	-0.37	-1.13	0.93	-0.21	-0.61	-0.85	-1.20	0.18	0.02
1.346	-0.17	-0.20	-0.75	-0.45	1.41	-0.31	-0.34	-0.17	0.10	-0.15	-0.25
1.700	0.08	-0.51	-0.77	0.37	1.26	-0.05	0.22	0.21	0.39	-0.01	-0.15
2.104	0.08	-0.55	1.10	0.78	0.82	0.17	0.46	0.68	0.93	0.11	0.15

The table highlights 8 (out of 199) data points that are outside the specification.

The Woods equation was also fitted to the following sets of calibration data for other small Ott Prop1 type meters:

3 calibrations for another small Ott

5 calibrations for a small Oss

2 calibrations for a Seba Mini

In each case the Woods equation fits the data quite closely, and the equation parameters are generally consistent with those found for the 10 runs for the small Ott meter, C2-170993.

The discrepancies between "true" and modelled velocity, using parameters fitted to data from each run are shown in Table 5. This gives us an impression of the error that would be committed by using this equation and fitting it to data from a single calibration run.

Table 5: Discrepancies between modelled and "true" velocities for each run (Small Ott C2-170993 10 runs)

Mean v	run1 Delta V%	run2 Delta V%	run3 Delta V%	run4 Delta V%	run5 Delta V%	run6 Delta V%	run7 Delta V%	run8 Delta V%	run9 Delta V%	run10 Delta V%	Mean Delta V%
0.023		3.21	-1.68	-5.79	-4.15	2.68	0.90	-0.06	0.85	-0.09	0.44
0.026	-2.09	-0.26	3.16	5.87	-5.39	0.56	-0.19	-0.03	1.93	4.39	0.42
0.030	3.20	-0.25	-3.54	1.54	1.02	1.81	-1.29	2.21	-0.64	-0.73	0.05
0.040	2.54	-4.65	1.28	-0.05	-1.14	-2.34	-0.85	-0.29	-0.94	-2.89	-0.97
0.050	-3.07	3.52	0.54	4.03	-2.20	-0.49	0.22	-3.08	-2.06	-1.63	-0.46
0.061	-8.54	-5.24	-5.11	6.15	1.20	3.16	2.56	1.75	0.89	3.43	0.13
0.070	0.34	-1.50	-2.30	0.71	0.76	1.52	-0.31	2.18	-1.61	-4.88	-0.43
0.081	3.04	-0.33	3.64	0.54	2.35	0.72	-1.02	-1.38	-1.00	1.06	0.85
0.091	0.14	2.06	1.67	3.07	2.73	1.00	-0.84	-0.70	-1.86	0.86	0.87
0.101	-4.28	2.93	1.43	-0.60	1.67	1.32	0.64	0.16	-0.85	2.09	0.50
0.148	-2.09	1.62	-0.83	-0.65	-1.80	2.93	1.30	1.13	0.55	1.20	0.26
0.198	-0.87	1.79	-0.61	-3.27	-2.36	1.56	-0.27	0.49	0.20	-1.44	-0.61
0.269	3.08	-2.07	-1.83	0.70	-4.68	-2.78	-0.70	-0.78	-0.56	-1.32	-1.16
0.348	0.87	-2.62	-1.75	0.96	0.67	-2.09	0.19	0.11	1.86	0.82	-0.02
0.451	-1.73	-0.56	0.91	0.69	-2.93	1.30	0.87	0.14	0.53	0.22	0.19
0.650	-0.66	1.35	2.24	-0.50	-0.97	1.39	-0.37	-0.62	-1.03	-0.11	0.43
0.951	0.97	1.56	-0.68	-2.08	0.68	-0.29	-0.56	-0.57	0.29	-0.61	0.02
1.346	0.06	-0.34	-1.04	-0.72	1.64	-0.17	-0.50	0.10	0.39	-0.45	-0.25
1.700	0.08	-0.24	-0.77	0.64	1.19	-0.04	-0.16	0.17	0.39	-0.06	-0.15
2.104	-0.05	-0.53	1.23	0.71	0.89	0.17	0.49	0.74	0.97	-0.04	0.15

The table highlights 15 (out of 200) data points that are outside the specification.

2.2. Reed Switch Pygmy

Extended range calibration data were for 10 calibration runs for the reed switch Pygmy meter: RS P 043 RS. The nominal velocity values for the 21 points for each calibration range from 0.04 – 2.1 m/s. Figure 4 and 5 show plots of the deviation of velocity from a straight-line relationship; Figure 4 shows the full range of data and Figure 5 shows only the lowest rotation rates.

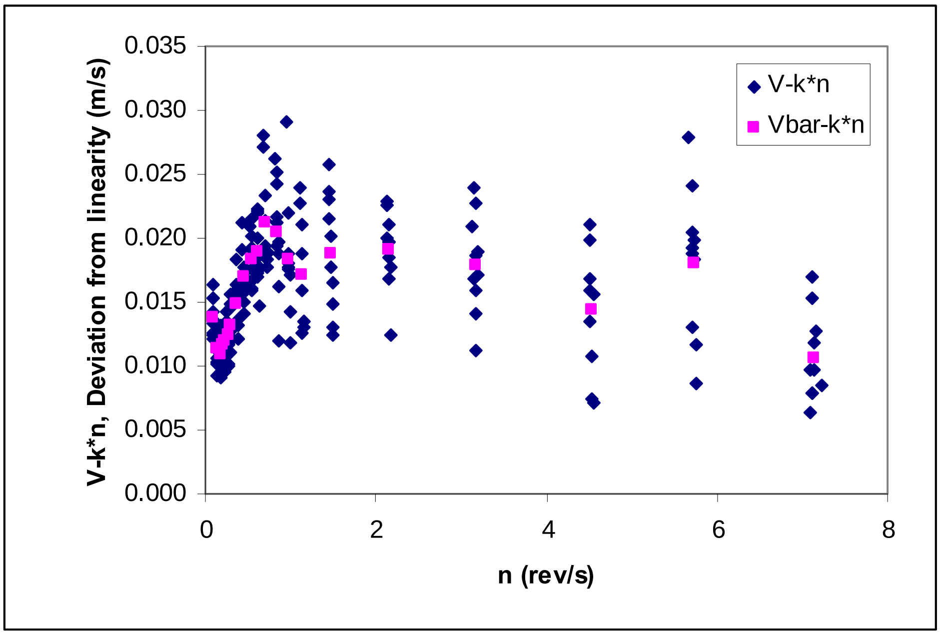

Figure 4: Velocity deviation from linearity (V-kn) plotted against rotation rate, for reed switch Pygmy meter.

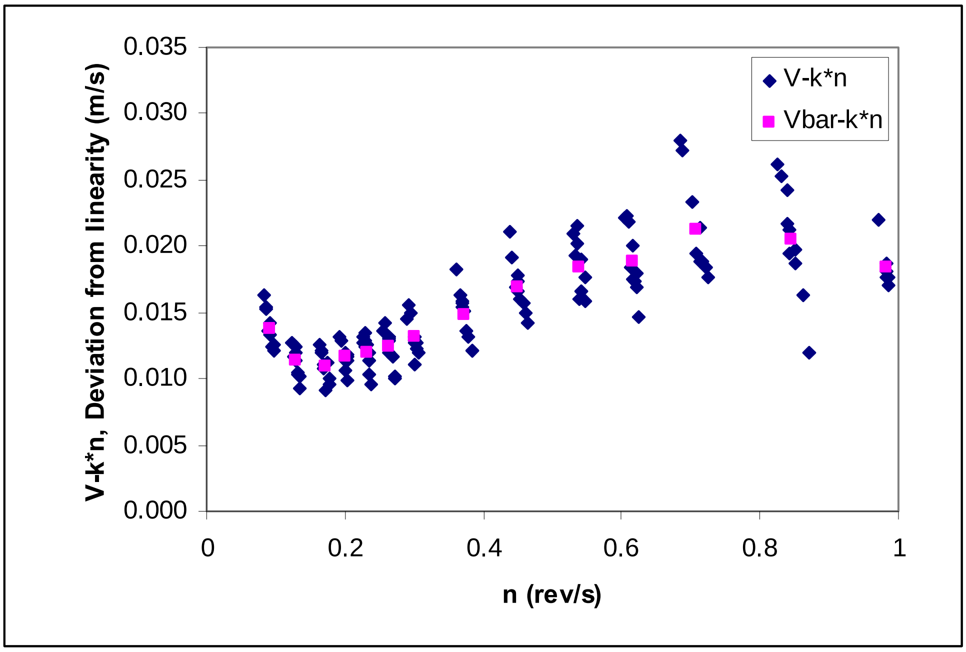

Figure 5: Velocity deviation from linearity (V-kn) plotted against rotation rate (low speeds), for reed switch Pygmy meter.

To represent the character of the Pygmy calibrations a 6-parameter Woods equation was developed to capture the unusual shape of the V-V' curve at low rotation rate Figure 5:

$$V = k\, n + v_0\, exp(-n/n_0) + a\, (abs(n-n_{low})/n_0)^p\, exp(-n/n_0)$$

This equation has one extra parameter, $\boldsymbol{n_{low}}$. Parameter values estimated for the 10 calibration runs are shown in Table 6.

Table 6: Optimised parameter values for each calibration run (Pygmy RS P 043 RS 10 runs)

Parameters	run1	run2	run3	run4	run5	run6	run7	run8	run9	run10
k	0.298	0.297	0.297	0.297	0.296	0.297	0.297	0.296	0.295	0.296
v0	0.011	0.013	0.017	0.015	0.015	0.014	0.017	0.010	0.008	0.012
n0	0.723	0.385	0.431	0.300	0.456	0.705	0.347	1.442	1.314	1.538
a	0.049	0.063	0.035	0.043	0.039	0.064	0.050	0.044	0.040	0.042
p	1.064	1.862	1.901	1.817	1.431	1.290	1.689	0.796	0.615	1.045
nlow	0.149	0.158	0.114	0.145	0.159	0.150	0.144	0.160	0.154	0.154

The equation fits each of the 10 runs quite closely. The parameter values are all quite different to those found for the small Ott Prop 1 type meters. Runs 8-10 have anomalous values for parameter n_0.

The discrepancies between measured and modelled velocity for each run are shown in Table 7. This table can be compared to Table 2, which highlights the limitations of the current limited range NIWA calibration equations for this meter type. No consistent evidence of bias is seen across the extended velocity range in the results for the Woods equation. The Woods equation provides fits where almost all points for all runs are within ± 2%.

The discrepancies between "true" and modelled velocity, using parameters fitted to data from each run are shown in Table 8. With a small number of exceptions the Woods representations of the individual calibrations agree with the "true" velocities to within 2%.

Table 7: **Discrepancies between modelled and measured velocities for each run (Pygmy RS P 043 RS 10 runs). A few anomalous data points for runs 1 and 2 have been removed (blank cells in table).**

Mean v	run1 DeltaV %	run2 DeltaV %	Run3 DeltaV %	run4 DeltaV %	run5 DeltaV %	run6 DeltaV %	run7 DeltaV %	run8 DeltaV %	run9 DeltaV %	run10 DeltaV %	Mean DeltaV %
0.040	-0.46	-0.34	-0.05	-0.12	-0.16	-0.27	-0.51	0.56	-0.62	0.23	-0.18
0.049	0.80	1.11	0.84	0.43	0.43	0.27	1.56	-0.76	-0.89	-0.19	0.51
0.061	-0.35		0.01	-0.03	0.05	-0.12	-0.50	0.15	0.53	0.01	-0.01
0.070	0.27	-1.08	-1.13	-0.38	-0.47	-0.44	-0.03	-0.16	0.89	-0.05	-0.12
0.080	0.22	0.07	-0.59	-0.41	0.59	0.46	-1.15	-0.52	-0.51	0.10	-0.07
0.090	0.88	0.60	1.07	0.59	-0.90	0.29	0.17	-0.07	-0.12	0.60	0.38
0.101	-0.12	-0.17	0.72	0.38	-0.05	-0.05	1.57	1.05	-0.29	0.33	0.39
0.124	0.53	0.72	0.66	1.04	1.65	-0.45	0.15	-0.61	0.85	-1.08	0.37
0.149	-0.84	0.58	0.74	-0.55	0.13	-0.82	0.45	-0.41	0.21	-0.63	-0.11
0.177	-1.27	0.08	-0.65	-0.32	0.11	0.65	0.09	-0.33	0.89	0.06	-0.10
0.200	2.76	0.04	-0.33	-0.20	-0.42	0.73	0.04	-1.35	0.08	-0.30	0.07
0.229	0.83	-1.43	-0.66	-0.55	-0.71	-1.42	-1.27	-0.54	0.08	-0.45	-0.65
0.269	0.63	-0.35	-1.00	1.25	0.14	0.36	-1.58	0.48	-0.97	-0.01	-0.17
0.307		1.02	0.27	0.71	0.60	-0.80	0.54	0.90	0.41	0.44	0.43
0.349	-1.11	0.38	1.34	-0.04	0.48	0.82	1.02	0.88	-0.70	1.20	0.32
0.451	-1.02	0.35	0.53	-0.69	-0.69	-0.15	-0.19	0.37	-0.96	-0.52	-0.47
0.649	-0.66	-0.97	-0.91	-0.71	-1.25	-0.39	-1.38	0.50	-0.75	-0.35	-0.91
0.947	-0.70	-0.58	-0.55	-0.14	-0.41	-0.64	-0.94	-0.17	-0.46	0.18	-0.53
1.341	0.01	-0.01	0.14	0.25	0.82	-0.24	0.09	0.01	-0.02	-0.08	0.16
1.697	0.14	0.63	-0.02	0.28	0.23	-0.14	1.07	-0.18	-1.25	-0.39	0.17
2.103	0.85		0.69	0.45	0.78	0.87	0.85	0.40	-0.03	0.37	0.73

3. Estimation of uncertainty

Uncertainty characterizes the range of values within which, with a given degree of confidence, the true value may be expected to lie. An uncertainty estimate assumes negligible uncorrected systematic error.

An estimate of the uncertainty of a current meter velocity measurement is required, without which the measurement is of little value.

In general a calibration process relates the magnitude of the output signal (the dependent variable) from a sensor, corresponding to a standard or reference (the independent variable). The reference itself will have a traceable relationship with the NZ and international standards.

Table 8: **Discrepancies between modelled and "true" velocities for each run (Pygmy RS P 043 RS 10 runs) A few anomalous data points for runs 1 and 2 were removed (blank cells in table).**

Mean v	run1 DeltaV %	Run2 DeltaV %	run3 DeltaV %	run4 DeltaV %	run5 DeltaV %	run6 DeltaV %	run7 DeltaV %	run8 DeltaV %	run9 DeltaV %	run10 DeltaV %	Mean DeltaV %
0.040	-2.09	-0.73	1.04	1.46	-0.55	-0.42	0.33	0.41	-1.02	-0.17	-0.18
0.049	0.26	-0.04	0.93	0.51	1.33	1.17	2.47	-0.68	-1.01	-1.33	0.51
0.061	-0.86		-0.50	0.77	2.82	-0.63	-1.01	1.94	-1.96	-0.83	-0.01
0.070	0.23	-5.07	-1.74	-0.42	0.77	-0.91	0.92	1.08	0.85	1.76	-0.12
0.080	0.20	-1.21	0.13	-1.18	1.19	-0.45	-0.43	0.07	-1.29	1.20	-0.07
0.090	0.77	0.37	1.52	0.48	-0.34	0.18	0.61	0.37	-1.45	0.60	0.38
0.101	-1.37	0.07	0.96	-0.88	-0.11	-0.21	1.91	1.69	0.63	0.67	0.39
0.124	0.72	0.18	0.76	1.48	1.75	-0.43	0.17	-1.15	1.28	-1.30	0.37
0.149	-1.21	0.07	1.31	0.08	0.29	-0.47	0.20	-0.32	-0.50	-0.60	-0.11
0.177	-0.05	0.35	0.52	-0.89	-0.30	0.30	-0.55	0.34	0.47	-0.92	-0.10
0.200	1.99	0.49	0.07	-0.20	-0.87	0.73	0.49	-0.85	-0.22	-0.60	0.07
0.229	0.65	-1.73	-1.48	0.19	-0.57	-1.76	-1.35	-0.01	0.21	-0.27	-0.65
0.269	0.21	-0.76	-1.12	1.09	0.09	0.28	-1.15	0.74	-0.83	0.39	-0.17
0.307		0.80	0.12	0.53	0.78	-0.69	0.26	1.01	0.65	0.62	0.43
0.349	-3.08	0.08	1.89	-0.14	1.17	0.32	1.25	1.19	0.15	1.45	0.32
0.451	-1.07	0.39	0.39	-1.16	-0.74	0.53	-0.15	0.34	-1.01	-0.50	-0.47
0.649	-1.13	-1.19	0.21	-0.71	-1.36	-0.85	-0.95	0.28	-0.59	-0.59	-0.91
0.947	-1.23	-0.30	-1.11	-0.68	-0.09	-0.12	-0.91	0.46	-0.79	0.37	-0.53
1.341	-0.39	4.19	-0.43	-0.33	-0.02	-0.48	-0.48	0.03	-0.63	-0.48	0.16
1.697	0.36	0.52	0.57	-0.07	-0.29	0.04	0.94	-0.32	-1.42	0.06	0.17
2.103	0.75	0.32	0.92	1.23	0.48	2.09	0.53	-0.21	-0.25	-0.13	0.73

In the case of a current meter calibration two references are required for velocity. i.e. distance and time. The time reference, a crystal clock, is used to estimate both the velocity of the car and the rate of rotation of the meter rotor.

In order to quantify the uncertainty of a field velocity measurement that corresponds to the variability of meter performance seen in the calibration process, we can think of a current meter calibration as the relationship between distance travelled by the car (and hence velocity) as the dependent variable, and the time taken for a given number of rotations (the independent variable). In this case the scatter of V values about a V vs n trend line, made using data collected at each nominal speed for multiple repeat calibrations, can be taken to be a representation of the variability of velocity attributable to meter performance.

The uncertainty in velocity for a given rotation rate can be estimated using the 10 repeated calibration points measured at each nominal speed setting. Each run is at a slightly different speed because the speed of the NIWA rating car is controlled by an analogue controller, but in every case the rotation rate and speed are very accurately measured.

Figure 6 shows 10 points at the same nominal speed (~0.124 m/s) from the 10 "Pygmy RS P 043 RS calibrations, and a trend line fitted to these data.

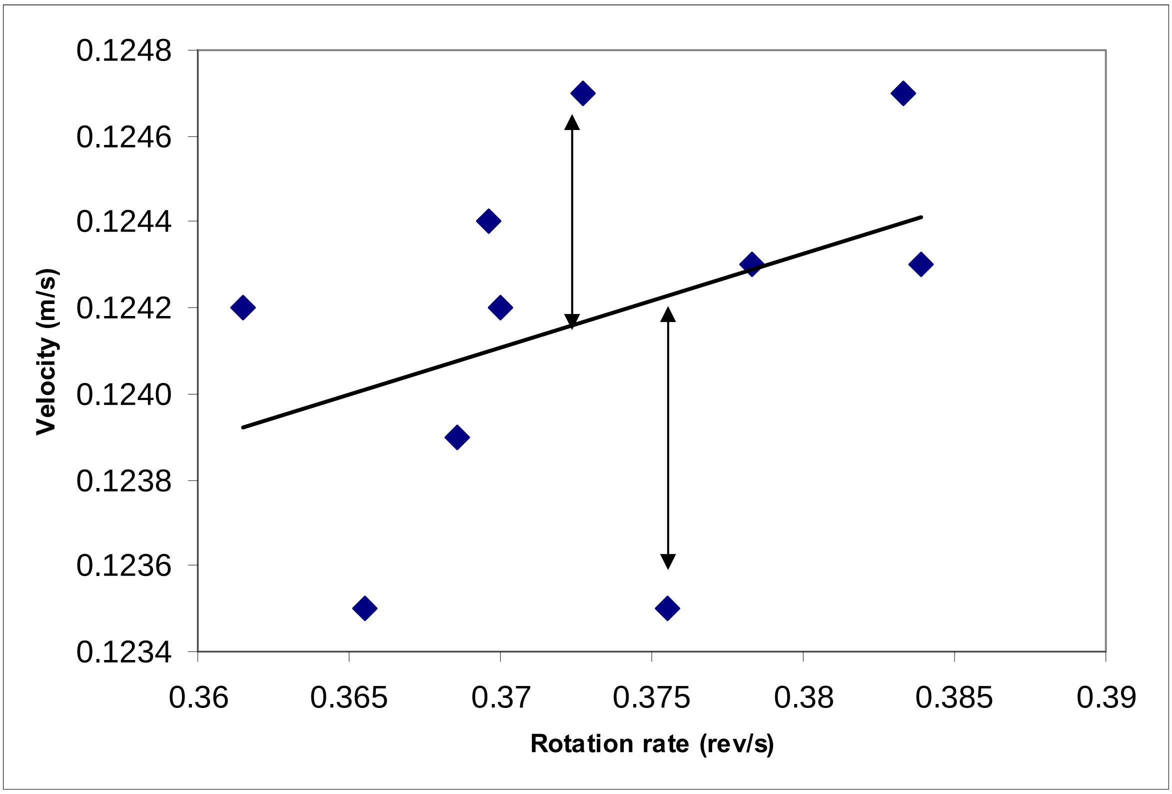

Figure 6: Variation in velocity about trend line fitted to 10-run calibration data for the nominal velocity of ~0.124m/s

The non-zero slope of the trend line (V = 0.022n + 0.116) explains some of the variation in velocity among the 10 runs. The remainder is ascribed largely to random variations in the meter performance.

The unexplained scatter of velocity values about the trend line is the experimental uncertainty in velocity, and we characterise this uncertainty using the standard deviation of the residuals around the trend line (arrows in Figure 6). This is the standard error of the regression, and is measured in units of velocity (e.g. m/s).

A regression line was fitted to every set of 10 calibration points for each nominal velocity, for both the Small Ott and Pygmy meters. Each standard error of the regression (labelled SigmaV in the Figures 7 and 8) was expressed as a percentage of the nominal velocity. The results for the two meter types are shown in Figures 7 and 8, and in Tables 9 and 10 respectively. Uncertainty at 95% confidence limits (estimated as 1.96 x SigmaV) for each nominal velocity is also shown in the tables.

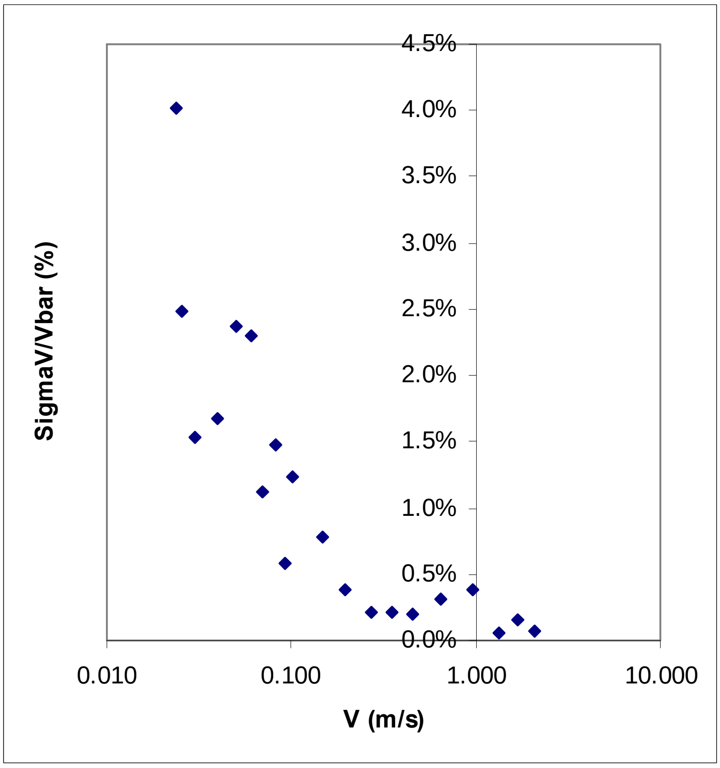

Figure 7: **Plotted Standard errors of regression for each of the 20 nominal velocities for 10 Small Ott C2-170993 calibrations**

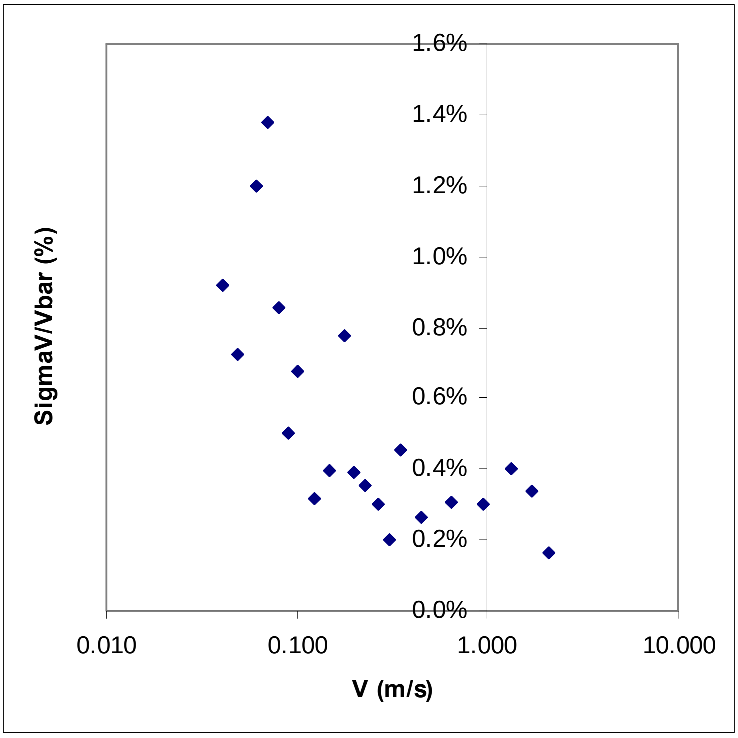

Figure 8: **Plotted Standard errors of regression for each of the 21 nominal velocities for 10 Pygmy RS P 043 RS calibrations.**

Table 9: Standard error of regression and uncertainty for each of the 20 nominal velocities for 10 Small Ott C2-170993 calibrations

Mean V m/s	SigmaV m/s	SigmaV/Vbar %	Uncertainty 95% confidence %
0.024	0.001	4.00%	7.84%
0.026	0.0006	2.50%	4.90%
0.03	0.0005	1.50%	2.94%
0.04	0.0007	1.70%	3.33%
0.05	0.0012	2.40%	4.70%
0.061	0.0014	2.30%	4.51%
0.07	0.0008	1.10%	2.16%
0.081	0.0012	1.50%	2.94%
0.091	0.0005	0.60%	1.18%
0.102	0.0013	1.20%	2.35%
0.148	0.0012	0.80%	1.57%
0.198	0.0008	0.40%	0.78%
0.269	0.0006	0.20%	0.39%
0.348	0.0008	0.20%	0.39%
0.451	0.0009	0.20%	0.39%
0.65	0.002	0.30%	0.59%
0.951	0.0036	0.40%	0.78%
1.346	0.0007	0.10%	0.20%
1.7	0.0026	0.20%	0.39%
2.104	0.0015	0.10%	0.20%

The uncertainty of velocity measurements attributable to the variability of the meter manifested by the 10 repeat calibrations across the velocity range is seen to meet the target specification, except for the lowest velocity

Table 10: Standard error of regression and uncertainty for each of the 21 nominal velocities for 10 Pygmy RS P 043 RS calibrations.

Mean V m/s	SigmaV m/s	SigmaV/Vbar %	Uncertainty 95% confidence %
0.04	0.0004	0.90%	1.76%
0.049	0.0004	0.70%	1.37%
0.061	0.0007	1.20%	2.35%
0.07	0.001	1.40%	2.74%
0.08	0.0007	0.90%	1.76%
0.09	0.0005	0.50%	0.98%
0.101	0.0007	0.70%	1.37%
0.124	0.0004	0.30%	0.59%
0.149	0.0006	0.40%	0.78%
0.177	0.0014	0.80%	1.57%
0.2	0.0008	0.40%	0.78%
0.229	0.0008	0.40%	0.78%
0.269	0.0008	0.30%	0.59%
0.307	0.0006	0.20%	0.39%
0.348	0.0016	0.50%	0.98%
0.451	0.0012	0.30%	0.59%
0.649	0.002	0.30%	0.59%
0.947	0.0029	0.30%	0.59%
1.347	0.0054	0.40%	0.78%
1.697	0.0057	0.30%	0.59%
2.103	0.0034	0.20%	0.39%

The uncertainty of velocity measurements attributable to the variability of the meter manifested by the 10 repeat calibrations across the velocity range is seen to meet the target specification for the full range.

4. Conclusions

The lowest velocity that can be measured by a small Ott Prop 1 type meter is approximately 0.024 m/s.

The lowest velocity that can be measured by a reed switch Pygmy meter is 0.04 m/s.

The Woods equation (5 and 6-parameter versions respectively) provides a good solution to the requirement for a function that represents extended range calibrations for both the small Ott Prop1 type meters and reed-switch Pygmy meters. Equations fitted to individual calibration runs show no evidence of consistent bias. Although the calibration data for the 10 calibration runs for the two types of meter were not screened when the data was collected, discrepancies between the true and modelled velocities exceed 2% for a small number only of the 200 data points for each meter. More sophisticated filters are planned to highlight an outlier point as a calibration proceeds. An anomalous point will be discarded and the measurement repeated. The forms of the Woods equations were tailored to capture the character of the two meter types.

The calibrations include points for the lowest velocities capable of sustaining meter rotation, and there is no need to extrapolate the calibration equations downwards. Alleged field measurements of velocities lower than the lowest calibration points, are assumed to have been achieved by sustaining the meter rotation by “jiggling” the meter. The magnitude of the actual velocity at these points can be said to be lower than the lowest calibration point for the meter type, but otherwise not quantified.

The new calibration equations will be made available electronically (probably on the NIWA web site) so that they can be loaded to the gauging calculation program “Gauge” or gauging computers without having to be keyed in.

The bias-free uncertainty of velocity measurements made using both meter types is within the target specification (± 5% for velocities less than 0.25m/s, and ± 2% for velocities greater than or equal to 0.25m/s) across the extended velocity range, with the exception of small Ott Prop1 meter measurements below 0.026 m/s.

5. Recommendations

Recommendations arising from this work are:

That the Woods 5 parameter equation be adopted to represent calibrations for small Ott Prop 1 type meters for the velocity range 0.025 – 2.10 m/s.

That the Woods 6 parameter equation be adopted to represent calibrations for reed switch Pygmy meters for the velocity range 0.04 – 2.10 m/s.

Tasks required to enable adoption of the new equations:

- determine the minimum set of calibration points that will represent calibrations for each meter type without introducing bias, or increasing uncertainty

- provide for accessing equations via the NIWA web site

- adapt the "Gauge" program for use of the new equations

- modify the calibration certificates to display equations of the new forms, and uncertainty at each velocity point.

    - modify calibration certificate to display uncertainty across range – probably as a column in a table of the actual calibration points.

6. Acknowledgements

The work was funded by an Envirolink medium advice grant. The author would like to thank Alistair McKerchar for searching for possible functions to describe the non-linear current meter calibration equations and Ross Woods for developing the fifth and sixth parameter version solution for the function.

Appendix 1:

Current meter calibration equations: Background Information

A1. Introduction

Relevant terms used in the science of *metrology* (measurement) are defined here, and some details of current meter characteristics and behaviour are described. Symbols and units for quantities are defined.

A2. Definitions, symbols and units

A2.1 Calibration

A definition of calibration is “a set of operations that establish, under specified conditions, the relationship between the values of quantities indicated by a measuring instrument, and the corresponding values realised by standards”.

In common with all measuring devices current meters need periodic calibration to monitor changes in performance.

The standards for a rating car used for current meter calibration are a quartz clock and the circumference of a reference wheel.

A2.2 Traceability

Confidence in the results of a calibration and their relationship to other current meter measurements demands that measurements have *traceability*.

The International Vocabulary of Basic and General Terms in Metrology defines traceability as:

"the property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of calibrations all having stated uncertainties."

Only the result of a measurement or the value of a standard can be traceable, not measuring equipment or a standard. What we mean when we talk about traceable equipment is that it is potentially able to produce traceable measuring results.

A2.3 The quality of standards used for a calibration

It is generally recommended that the measurement standards against which calibrations are made should be about four times better than that expected of the instrument being calibrated.

A2.4 Propagation of errors through the unbroken calibration chain

Traceability of measurement provides the best estimate for the value (e.g. velocity), but each step in the chain introduces some additional uncertainty. This is because each instrument in the chain is subject to possible drift, limited resolution, sampling uncertainties. The uncertainty at any point in the chain includes the accumulative result of all preceding uncertainties. The specification for a properly traceable instrument used for a standard will include all “bought-in” calibration uncertainty.

A2.5 Error and uncertainty

The error in a velocity measurement has distinct random and systematic components.

It is likely that personnel carrying out current meter gaugings will be unaware of the magnitude of any systematic error due to a calibration equation misrepresenting the calibration data, and this error will not be identified or quantified with the gauging result. The systematic error cannot be reduced by repeat field measurements or longer sampling times.

Random error associated with current meter performance is quantifiable and is independent of the fitted calibration equation. Random error can be reduced by repeat measurements. The uncertainty of a velocity result estimated as the average of a set of measurements will be smaller than that consisting of a single measurement. ISO 7066:1989 calls for accuracy tolerance limits for current meter ratings to be at the 95% confidence level, which is about 2 standard deviations from the mean.

When reporting the result of a measurement of a physical quantity, it is obligatory that some quantitative indication of the quality of the result to be given so that those who use it can assess its reliability. Without such an indication, measurement results cannot be compared.

In general, a measurement has imperfections that give rise to error in the measurement result. Errors have two components, a **random** component and a **systematic** component.

Random errors result from variations in the measurement process due to small changes in the instrument, or from electrical noise. **Precision** is a measure of the spread of repeated measurements and is specified by the **standard deviation** of the repeated readings.

Bias refers to a consistent deviation in measured values from the true value, caused by systematic errors. **Systematic errors** arise from errors in theory, calibration, imperfections in the apparatus, imperfect observations, false assumptions etc.

Measurement **uncertainty** is a parameter associated with a result that describes the region about an observed value that is likely to enclose the true value.

Confidence limits are the upper and lower limits about an observed or predicted value within which the true value is expected to lie with a specified probability, assuming negligible uncorrected systematic error.

A2.6 Velocity

Velocity, V (m/s) is velocity of meter relative to water

Rotational rate, n (revs/s) of meter

Pitch, k (m/rev) refers to the water translation per revolution of the meter, and corresponds to the ratio V/n.

Sensitivity, **S** (revs/m) refers to the output of the meter in terms of revs per metre of water translatuion, and corresponds to the ratio n/V. Sensitivity is the reciprocal of pitch.

The quantities pitch and sensitivity also apply to boat propellers and the threads of wood or machine screws. A coarse thread, analogous to a small Ott prop 3 has a large pitch (m/rev) and a small sensitivity (revs/m). A fine thread, analogous to a small Ott prop1 has a small pitch (m/rev) and a large sensitivity (revs/m). Meters with high sensitivity (many revolutions per metre of water translation) are better for audibly detecting pulsating velocity.

Although vertical axis (bucket) meters do not resemble propellers or screws, the quantities pitch and sensitivity also apply to them.

A comparison of sensitivity and pitch values for a variety of current meters at a velocity of 2 m/s is shown in Table A1.

Table A1: Sensitivity and pitch for common meters at a velocity of 2 m/s.

Meter		Sensitivity (revs/m)	Effective pitch (m/rev)
Price AA (Read Switch model)		1.5	0.68
Small Ott	Prop 1	18.4	0.05
	Prop 3	4.0	0.25
	Prop 5	18.3	0.06
Large Ott	Prop 1	4.0	0.25
	Prop 2	2.1	0.48
Large Seba	Prop 1	3.0	0.33
Pygmy (Read Switch model)		3.4	0.30
Amsler		3.3	0.31

A current meter **calibration** is carried out to determine the rotation rate, n, for a series of car velocity settings. The results are plotted with velocity on the Y axis and rotation rate on the X axis.

A calibration line for a wood screw is a straight line passing through the origin (Figure A1). In this case the pitch of the screw is 0.1 m/rev.

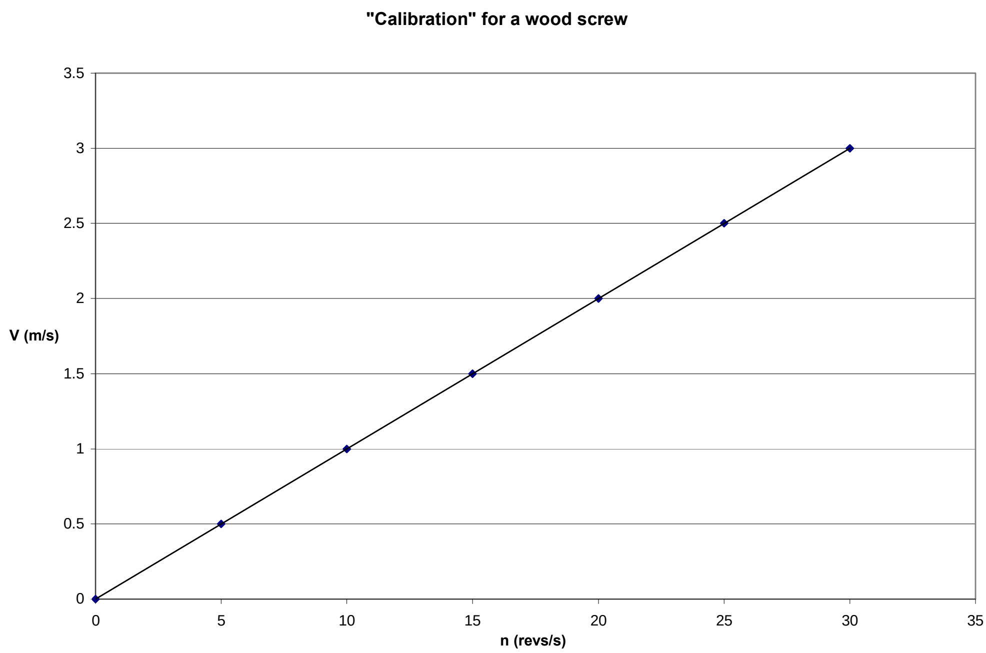

Figure A1: Calibration line for wood screw.

A wood or machine screw has a constant pitch and sensitivity i.e. metres per rev, and revs per metre are constant across the velocity range. A plot of sensitivity v velocity for the wood screw calibration above is shown in Figure A2.

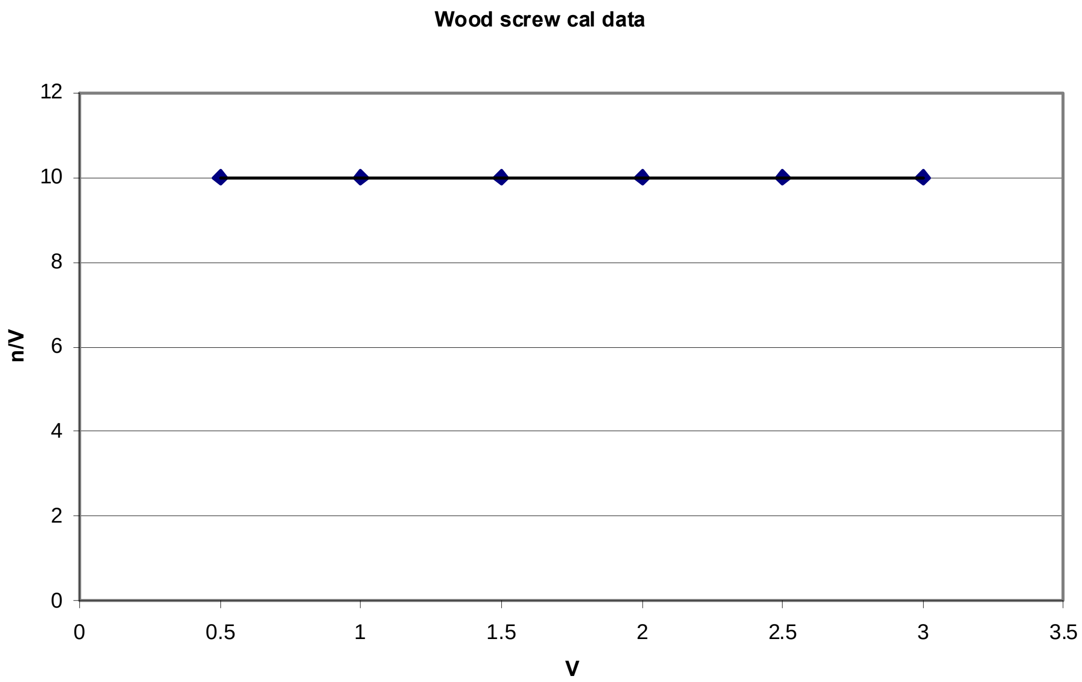

Figure A2: Sensitivity plot for the wood screw calibration shown in Figure 1.

An ideal current meter, without inertia, friction, or switch detention would behave like a wood screw or machine screw. However, particularly at low velocity, current meters do not "engage" fully with water. At very low velocity the pitch (m/rev) manifested by a current meter is high compared to the theoretical "design" pitch, and the sensitivity or output (revs/m) lower than the theoretical sensitivity. As velocity rises, the manifested pitch decreases and the sensitivity increases, and some meters eventually attain constant, theoretical values for pitch and sensitivity. The velocity range where the pitch is high and sensitivity is low compared to the theoretical values can be called the *sub-screw state*, and the range where theoretical, constant values for the two quantities are attained can be referred to as the *screw state*.

A schematic calibration for a real current meter is shown in Figure A3. The curvature of the low-velocity sub-screw region is exaggerated.

The difference between the observed car velocity, V, and the corresponding "theoretical" (constant pitch meter) velocity, V', for a given rotation rate, becomes more obvious when the difference, V-V' is plotted against n or V for a set of calibration points. Figure A4 shows how V-V' varies with V for a set of small Ott Prop 1 calibration data.

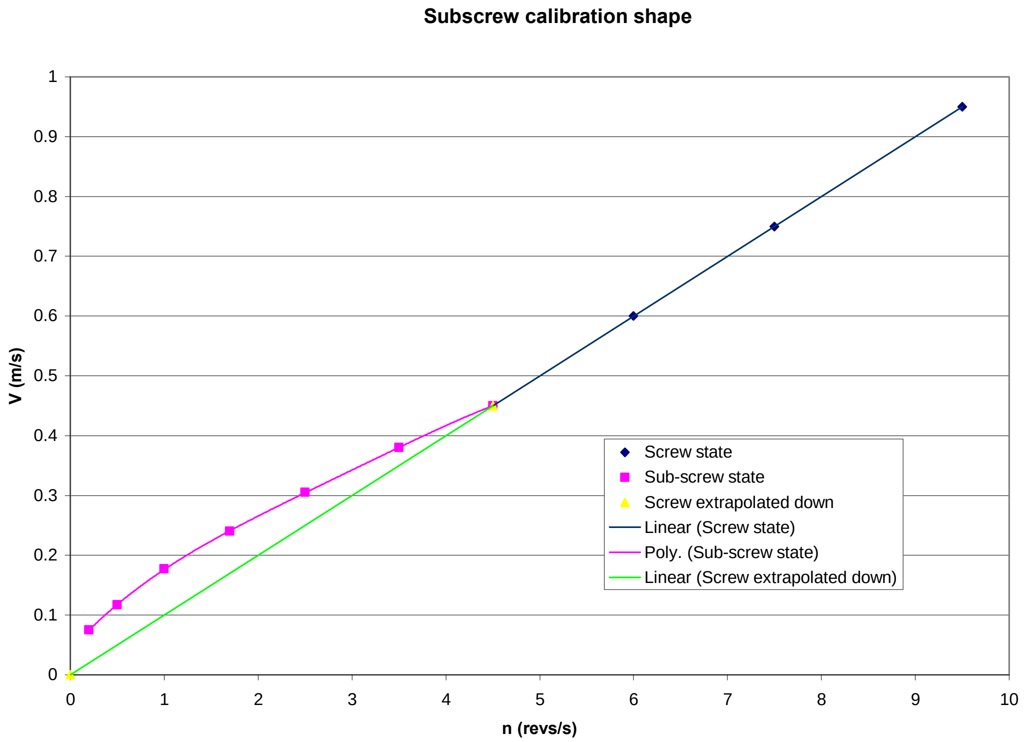

Figure A3: **Schematic depiction of low velocity non-linear sub-screw state**

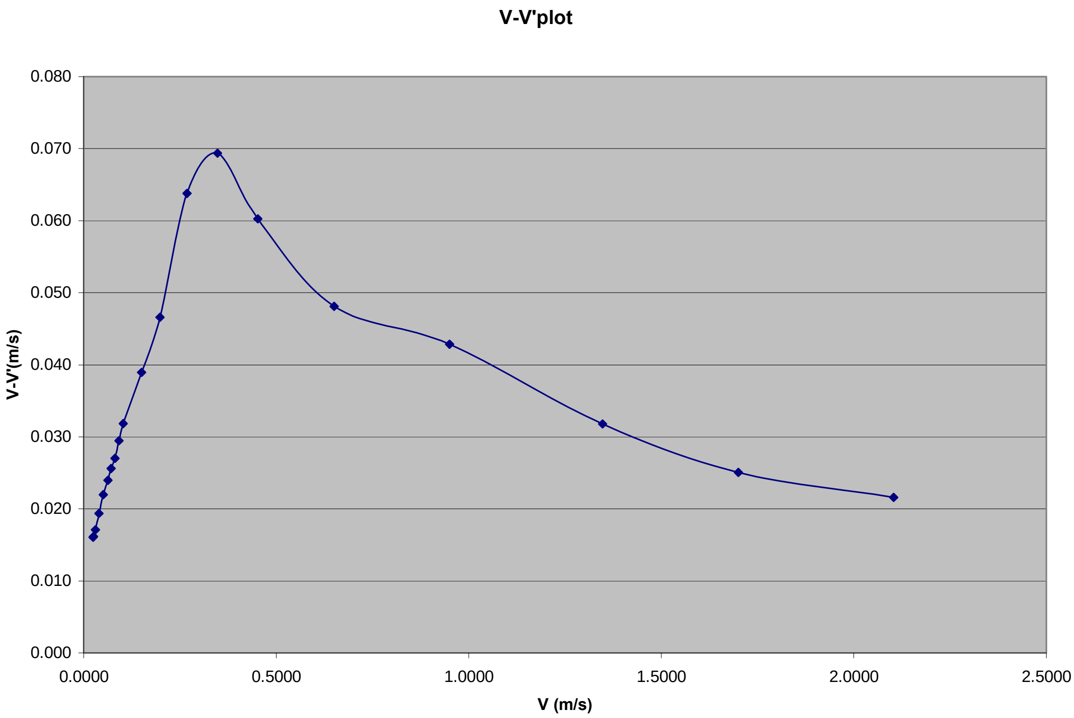

Figure A4: **V-V’ plot for a small Ott prop1 meter.**

The differences between real and ideal meters velocities are seen to reach a maximum at approximately 0.35 m/s for this meter.

The change in meter sensitivity (or meter output) with velocity for the calibration of a small Oss prop1 meter is shown in Figure A5.

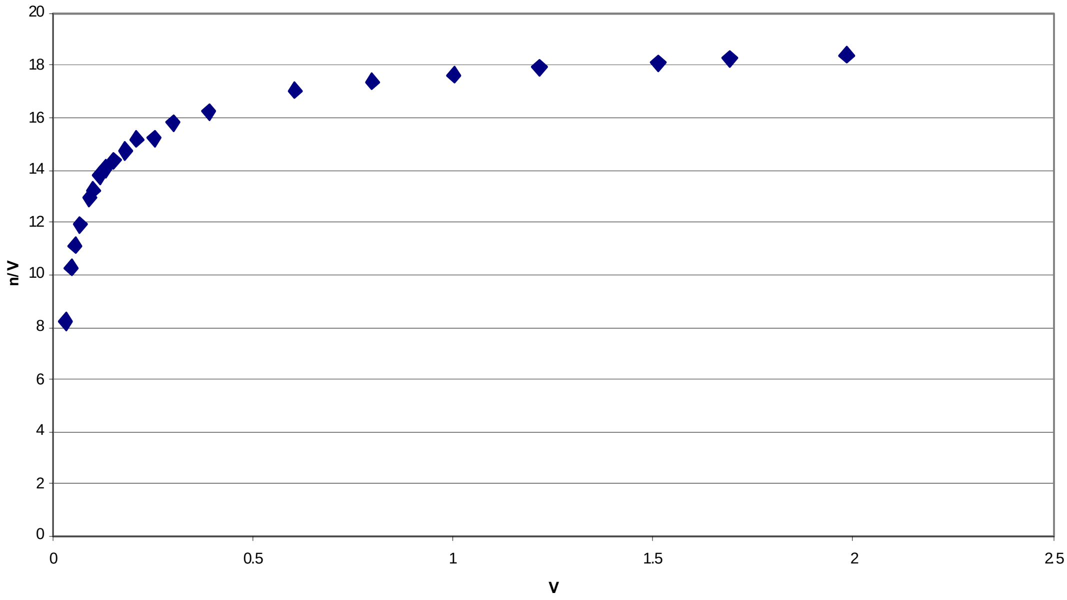

Figure A5: Sensitivity curve for an Oss prop1 meter

A sensitivity plot for a small Ott prop 3 is shown in Figure A6. This meter does attain a screw state. The sensitivity (and pitch) becomes constant at velocities greater than 0.1 m/s.

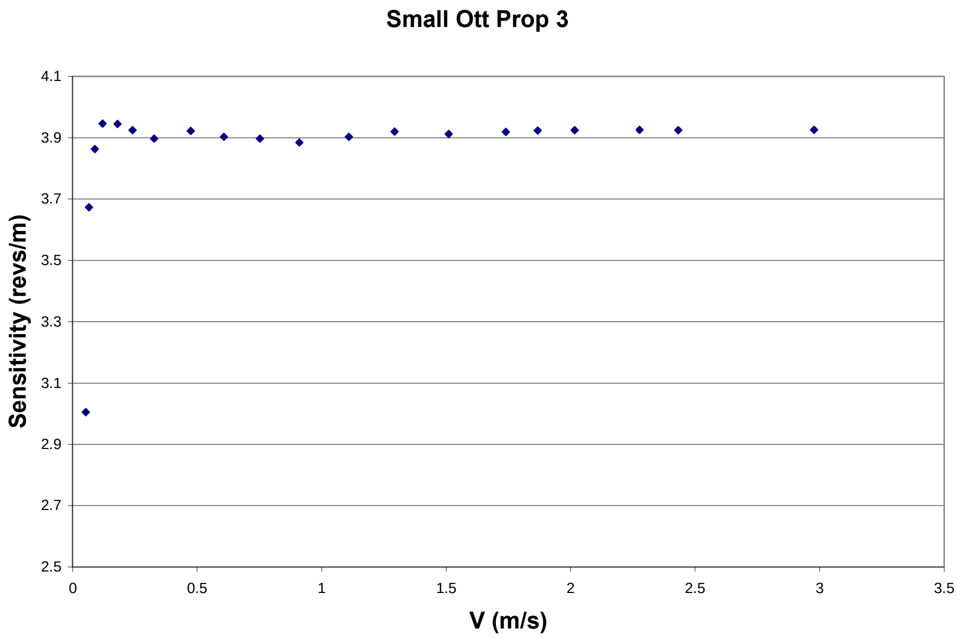

Figure A6: Sensitivity curve for small Ott Prop 3

A3 Current meters used in New Zealand

An approximate inventory of the New Zealand pool of current meters is shown below.

Meter type		Number
Price AA	Wiping contact	44
	Reed switch	115
Pygmy	Wiping contact	83
	Reed switch	51
	Magnetic head	6
Small Ott C1	Wiping contact	31
Small Ott C2	Reed switch	48
Seba mini		13
Small OSS PC1		8
Large Ott C31		63
Seba Univaersale		11
Large Oss B1		2
Amsler		23

The small Ott C1 (wiping contact) meters are an older model than the C2 (reed switch) meters and are gradually being retired. All the small Oss PC1 and Seba Mini meters in use are reed switch meters.

Magnetic Head Price Pygmy meters are fitted with miniature reed switch and magnet components. These meters will rotate at lower velocities than their Reed Switch counterparts.

ISO 748 stipulates that the minimum water depth allowable for a current meter measurement depends on the rotor height: “the horizontal axis of the current meter shall not be situated at a distance less than 1.5 times the rotor height from the water surface, nor shall it be at a distance less than 3 times the rotor height form the bottom of the channel.” Accordingly the minimum depth recommended for Pygmy and small Ott measurements are 0.15 and 0.38 m respectively.

A4 Meter output dependency on variables other than velocity

Meter output varies with velocity, but also with water temperature, and with the diameter of the mounting rod, and the distance between the propeller and the rod. The output of a meter, in terms of revolutions per metre, is less for a larger diameter rod, and when the meter is mounted closer to the rod. The meter output in a tow tank also depends on the depth of the meter below the water surface and the length of rod protruding below the meter. Calibration data are not directly comparable, if any of these additional parameters differs for the calibrations being compared. Meter performance can also vary with the tank water temperature.

It is of course preferable that the rod diameter and relative position to the meter are the same at the time of calibration as will be used for gauging, and agencies sending meters to NIWA for calibration specify the rods used. Most small Ott and Seba meters are used with 0.5 inch diameter rods, as supplied by NIWA. Hydrological Services supply 9mm diameter rods with small Oss meters and the 9mm rods are often used for gaugings using small Oss meters. The meters can also be used with 0.5 inch rods. The clamping arrangement for the 9mm rods is such that the distance between the prop and the rod is closer than is the case where the 0.5"rods are used. Consequently, to an extent the effects of the proximity of the rod and the rod diameter cancel each other.

Most meters calibrated at the NIWA tank are mounted at a depth of 300mm with 100mm of rod protruding below the meter.

Meter output is higher for higher water temperature. This is attributed to reduced oil viscosity at higher temperatures, but may be partly caused by small changes in component dimensions due thermal expansion.